

## MODELING SOIL THERMAL DIFFUSIVITY AS A FUNCTION OF SOIL MOISTURE

Soil thermal parameters are playing an important role in soil heat transfer. Mathematical models and Pedotransfer functions (PTFs) are considered the modern soil physics. Thermal diffusivity is an important parameter of the soil thermal properties, used to study the soil heat flow. Recently, PTFs are widely used to forecast thermal properties. The aim of the work is to ascertain the relation between thermal diffusivity and soil moisture by a model. The relationship between thermal diffusivity and soil moisture by is described by a quadratic equation, and then determined the parameters of this equation using PTFs. The determination coefficient (R<sup>2</sup>), Root Mean Square Error (RMSE) and Willams-Kloot test exhibited that the proposed PTF of the quadratic equation is better than Arkhangelskaya (2004) model for predicting soil thermal diffusivity as a function of soil physical properties.

**Key words:** soil physical properties, thermal parameters, Soil moisture, pedotransfer functions (PTFs), mathematical models.

### Introduction

Soil thermal parameters are mainly input for modeling of soil heat flux. Mathematical models and PTFs are important tools for predicting soil heat transfer. PTFs are depended on soil physical characteristics (particle size distribution, bulk density and organic matter). Thermal diffusivity is one of the important soil thermal parameters. Thermal diffusivity

$$K = \frac{\lambda}{c \cdot v}$$

is defined as the ratio of the thermal conductivity ( $\lambda$ ) to the volumetric heat capacity ( $Cv$ ) [5]. It is a parameter that quantifies the ability of soil to store thermal energy during soil heat transfer processes. Thermal diffusivity is the controlling in soil heat flux during transient conductive heating processes. The literature contains many studies the influence of soil moisture content, bulk density, tillage system, salt concentration, and organic matter on soil thermal parameters [1]-[3]. There are many methods used to measure thermal diffusivity, which may be divided into two types: (1) direct methods have been measured in the laboratory according to Kondratieff [5] are more accurate however, costly and time-consuming and (2) indirect methods based on mathematical models and Pedotransfer functions (PTFs), it is depending

on describe the relationship between soil thermal diffusivity and soil moisture by a model, then estimate the parameters of this model by PTFs. PTFs estimate soil thermal parameters based on various physical and chemical soil characteristics. Particle size distribution, soil bulk density and soil organic matter are commonly used predictor variables in PTFs.

There are many mathematical models study soil thermal parameters by defining equation, such as a model of Chung and Horton [3] describe the relation between soil thermal conductivity and soil moisture

$$\lambda \cdot (\theta) = b_1 + b_2\theta + b_3\theta^{0.5}.$$

The empirical parameters  $b_1$ ,  $b_2$  and  $b_3$  are depended on soil physical properties. HYDRUS1-D program can be estimated those parameters depend on soil texture.

On the other hand, Tikhonravova and Khitrov [6] estimated soil thermal diffusivity as a function of soil moisture by polynomial equation,

$$K = K_0 + a_1\theta + a_2\theta^2 + a_3\theta^5.$$

Where,  $K_0$ ,  $a_1$ ,  $a_2$  and  $a_3$  are the parameters of the equation. Moreover, Arkhangelskaya [1] suggested another kind of model: a lognormal equation dependence on thermal diffusivity from soil moisture. She described the relationship

between thermal diffusivity and water content by S shape curve.

$$K = K_0 + a \exp \left[ -0.5 \left( \frac{h \left( \frac{w}{w_0} \right)}{b} \right) \right]$$

Where,  $w$  is the water content,  $K$  is the corresponding thermal diffusivity;  $K_0$ ,  $a$ ,  $w_0$ , and  $b$  are parameters of the curve:  $K_0$  is thermal diffusivity of dry soil,  $w_0$  is the water content corresponding to the maximum thermal diffusivity, and  $(K_0+a)$  is the maximum thermal diffusivity at  $w=w_0$ . The parameter  $b$  characterizes the peak width of the curve. However, this model includes many (four) parameters. So, it is important to calculate soil heat transfer, and thermal diffusivity without data, is to estimate a model describes the relation between thermal diffusivity and soil physical parameters using PTFs. The objective of this work is to propose a mathematical model that will allow to estimate the thermal diffusivity from soil water content based on soil physical properties, and the tasks were several: (1) to measure soil thermal diffusivity under different values of soil moisture by the direct method; (2) to describe a relation between thermal diffusivity and soil moisture by a model; (3) to estimate parameters of this model based on soil physical characteristics using PTFs and (4) compare computed  $K$  that obtained by the proposed PTF with the Arkhangel'skaya PTF [1] to estimate a validation of this model.

### Material and methods

#### Soil samples

The silt loam and silt clay loam soil representing major Agriculture in sod-podzolic soils of the Moscow region, Russia, Zelenograd field laboratory of Soil Science Institute. Four soil profiles were dug and, undisturbed soil samples were collected according to the difference in the depth. Soil samples (13 samples) were collected and soil thermal diffusivity was measured under different values of soil moisture (7 levels for each sample),  $N=91$  repeats. PTF was estimated based on soil physical properties. Three soil profiles were dug down, to determine and propose PTFs. Moreover, one soil profile was dug down to test and evaluates the efficiency of this PTF.

### Soil Physical Properties

Particle Size Distribution was measured using the pipette Method according to [5]. Soil bulk density (B.D.) was determined by core method, organic matter (OM), which was measured using high temperature using (Express analyzer AN-7529), soil particle density was determined by pycnometer according to [5].

### Experimental thermal diffusivity

One method for measuring thermal diffusivity  $K$  directly in the laboratory is depended on placing a heat source, having a constant temperature in contact with the surface of a soil column having constant cross-sectional area and insulated sides. Soil thermal diffusivity ( $K_{exp}$ ) was determined in the laboratory by the direct method using Kondratieff method according to [5]. Soil cores were initially saturated with water and then subjected to drying, thermal diffusivity was measured under different values of soil moisture. The levels of soil water content were 0.05, 0.15, 0.2, 0.25, 0.28, 0.35, and 0.4 g/g.

### Mathematical models

The model of Arkhangel'skaya [1] (KPTFs-2), and the proposed quadratic equation (KPTFs-1) were used to estimate thermal diffusivity as shown in the table (1).

1-The parameters of a model of Arkhangel'skaya,  $K_0$ ,  $a$ ,  $w_0$ , and  $b$  were determined from PTFs.

$$K = K_0 + a \exp \left[ -0.5 \left( \frac{h \left( \frac{w}{w_0} \right)}{b} \right)^2 \right];$$

$$K_0 = -1.06 + 1.98.B.D + 0.20.C;$$

$$a = -0.58 + 2.43.B.D - 0.08.C;$$

$$b = 0.12 + 0.12.B.D + 0.12.C.$$

Where,  $B.D$  is the bulk density ( $Mg/m^3$ ) and  $C$  (%) is the organic Carbone concentration.

2-The quadratic interpolation equation was to estimate thermal diffusivity  $K(\theta)$  as a function of soil moisture.

$$K(\theta) = b_1 + b_2\theta - b_3\theta^2.$$

The parameters of equation  $b_1$ ,  $b_2$  and  $b_3$  were estimated by PTFs based on soil physical properties [4].

### Statistical analysis

The efficiency of the Pedotransfer functions (PTFs) was determined using the determination coefficient ( $R^2$ ), Root Mean Square Error (RMSE) and Williams-Kloot test.

$$RMSE = \sqrt{\frac{\sum_{N} (Y_m - Y_c)^2}{N}} \cdot \sqrt{\frac{\sum_{N} (Y_m - Y_c)^2}{N}}$$

Where,  $Y_m$  is the measured value of  $K$ ,  $Y_c$  is the corresponding calculated.

### Williams-Kloot test:

$$\frac{(D1 + D2)}{2} = a \cdot (D2 - D1),$$

where,  $D1$  and  $D2$  denote the errors of the model calculated and experimental values,  $a$  - is a regression coefficient, if  $t$  value for  $t_a > 0$ , the first model is better, if  $t_a < 0$ , the second model is better.

### Software

Software tools were Microsoft excel, and SPSS program for the Statistical Analysis (Table.1).

## Results and Discussion

### 1-Modeling thermal diffusivity as function of soil moisture.

Variation of thermal diffusivity of silt loam and silt clay loam of soil samples as a function of soil moisture is shown in Fig.1. The thermal diffusivity values of soil samples were varied in the range

from  $1.21 \times 10^{-7}$  to  $2.417 \times 10^{-7}$  m<sup>2</sup>/s, the range of soil moisture was 0.05 to 40 g/g. Thermal diffusivity at first increased rapidly with increasing water content to reach the maximum, then decreased at a slower rate. The reason of that water content increase thermal contact between soil particles and replaces the air, which has lower thermal conductivity than water and increases volumetric heat capacity between soil partials sequence increasing soil thermal diffusivity. But the thermal diffusivity is increasing more rapidly than the volumetric heat capacity, as a result of that decrease thermal diffusivity. While soil thermal diffusivity increased liner with increasing soil bulk density. This result was agreed with [2]-[3].

We could describe the represented data in Fig.1: by  $\cap$  shaped curve using a quadratic interpolation equation.

$$K(\theta) = b_1 + b_2\theta - b_3\theta^2.$$

Where,  $b_1$ ,  $b_2$ , and  $b_3$  are the experimental parameters depended on soil physical properties and  $\theta$  is a fraction of mass soil moisture [4].

### 2-Parameterization and Pedotransfer functions (PTFs)

The parameters of a quadratic equation were estimated by the fitting curve using SPSS program. Take into account the relations between parameters of the quadratic equation and soil physical properties which more effect on soil thermal diffusivity. Table.2: shows that Correlations between soil physical properties (bulk density, organic matter and clay content) and parameters of the quadratic equation.

We could determine PTFs to calculate parameters of the quadratic equation  $b_1$ ,  $b_2$  and  $b_3$  as a function of soil physical properties using statistical regression as flowing equations:

Table.1: Different methods used to estimate thermal diffusivity

Thermal diffusivity	Model	Method	Parameter
$(K_{exp})$	In Lab (Kondratieff Methods)	Kondratieff Methods	
$(K_{PTFs-1})$	$K(\theta) = b_1 + b_2\theta - b_3\theta^2$	Quadratic equation	$b_1, b_2$ and $b_3$
$(K_{PTFs-2})$	$K = K_0 + a \exp \left[ -0.5 \left( \frac{\ln \left( \frac{w}{w_0} \right)}{b} \right)^2 \right]$	Arkhangel'skaya (2009) Lognormal equation	$K_0, a, w, w_0$ and $b$

$$K(\theta) = b_1 + b_2\theta - b_3\theta^2;$$

$$b_1 = 2.00 \cdot 10^{-7} \exp \cdot (-0.09 \cdot \text{B.D} - 0.035 \cdot \text{O.M} - 0.067 \cdot \text{CL});$$

$$b_2 = 2.46 \cdot 10^{-7} \exp \cdot (0.57 \cdot \text{B.D} - 0.079 \cdot \text{O.M} + 0.011 \cdot \text{CL});$$

$$b_3 = 2.53 \cdot 10^{-7} \exp \cdot (0.79 \cdot \text{B.D} - 0.107 \cdot \text{O.M} + 0.015 \cdot \text{CL}).$$

Where, (B.D) is bulk density (Mg/m<sup>3</sup>), (O.M) is organic matter content, and (CL%) is clay content (Table.2).

**Table.2: Correlations (R) between Bulk Density (B.D), Organic matter (O.M) and Clay content (CL) with parameters of a quadratic equation b<sub>1</sub>, b<sub>2</sub>, and b<sub>3</sub>**

Parameters	R		
	B.D	O.M	CL
b <sub>1</sub>	-0.509	0.684	-0.630
b <sub>2</sub>	0.879	-0.850	0.883
b <sub>3</sub>	0.799	-0.743	0.785

**The efficiency of PTFs for predicating thermal diffusivity.**

Table.4: shows that the calculated values of K obtained by two PTFs compared to the corresponding measured value of (K<sub>exp</sub>) by the direct method. The results observed that the best PTF could be used to calculate soil thermal diffusivity as a function of soil moisture was the quadratic equation (K<sub>PTFs-1</sub>), for silt loam and silt clay loam sod-podzolic soil. R<sup>2</sup> for (K<sub>PTFs-1</sub>) was higher than (K<sub>PTFs-2</sub>) were 0.873 and 0.730. Moreover RMSE

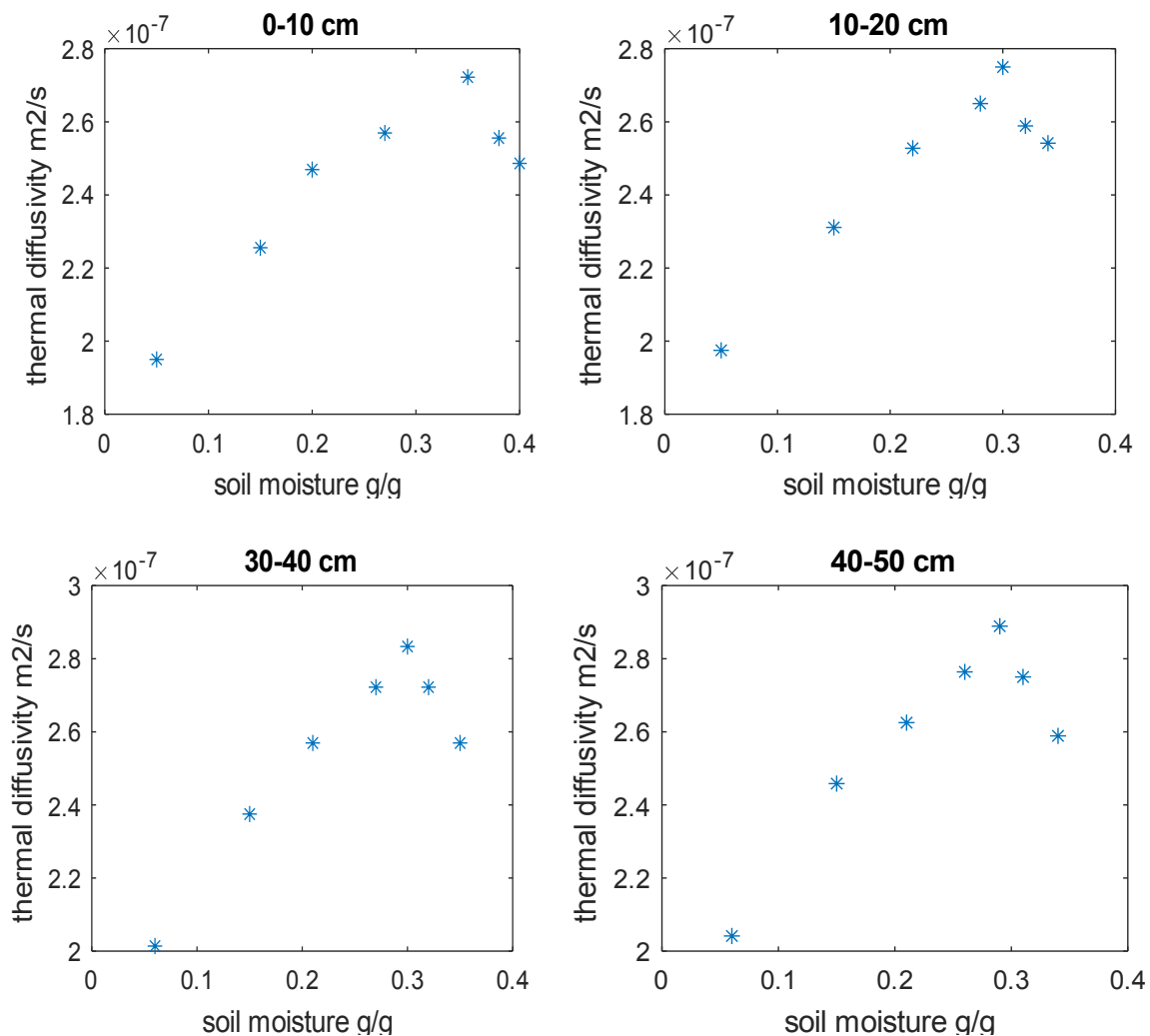


Fig.1: Thermal diffusivity (m<sup>2</sup>/s)at different values of soil moisture (g/g) for different soil depths.

for ( $K_{PTFs-1}$ ) was lower than ( $K_{PTFs-2}$ ) were  $8.8 \times 10^{-8}$  and  $1.4 \times 10^{-7}$ . The reason of that the model of Arkhangel'skaya [1] described the relationship between thermal diffusivity and water content by S-shaped curve. However, this description differs from ours as we described thermal diffusivity by  $\cap$  shaped curve that described represented data from experimental thermal diffusivity. Williams-Kloot criterion, demonstrated that Student test was 13.33 and was significant  $< 0.005$ . It means and ensures that the first PTFs of the quadratic equation is better than PTFs of Arkhangel'skaya [1] for predicting thermal diffusivity under silt loam and silt clay loam soil.

### Conclusion

Soil moisture and soil bulk density are soil physical properties have great effect on thermal diffusivity. The effect of soil bulk density and soil moisture are more than clay content on thermal diffusivity. Thermal diffusivity at first increased rapidly with increasing water content then decreased at a slower rate. We described the relationship be-

tween thermal diffusivity and soil moisture by  $\cap$  shaped curve using a quadratic equation ( $K_{PTFs-1}$ ). Moreover, estimate the parameters of this equation by PTFs. PTFs based on soil physical properties (particle size distribution, bulk density and organic matter). There are strong correlations between parameters of the quadratic equation ( $b_1$ ,  $b_2$  and  $b_3$ ) and soil physical properties (bulk density, organic matter content and clay content). As well as there are correlations between parameters of the quadratic equation that measured by fitting curve and by PTF:  $K_{PTFs-1}$  by quadratic equation and  $K_{PTFs-2}$  by the lognormal model of Arkhangel'skaya.  $R^2$ , RMSE, and Williams-Kloot criterion exhibited that ( $K_{PTFs-1}$ ) is better than ( $K_{PTFs-2}$ ). The PTF of the quadratic equation is the accurate equation for forecasting soil thermal diffusivity, for silt loam and silt clay loam sod-podzolic type of soils genesis.

### Acknowledgment

This work was supported by the Russian Scientific Fund (the project № 14-16-00065).

10.11.2017

### Reference:

1. Arkhangel'skaya T.A. Parametrization and mathematical modeling dependence of termodiffusivity from soil moisture. Pochvovedenie. 2009. № 2. С. 178–188. [in Russian].
2. Bouma, J. Using soil survey data for quantitative land evaluation. Advances in Soil Science. 1989. Vol. 9. Pp. 177–213.
3. Chung J., R.Horton. Soil heat and water flow with a partial surface mulch. Water Resour. Res. 1987. Vol.23. Pp. 2175-2186.
4. Shein, E.V., A.Y. Mady. Soil Thermal Parameters Assessment by Direct Method and Mathematical Models. Journal of Soil Science and Environmental Management. 2016.7 (10):166-172.
5. Theories and Methods of Soil Physics. 2007. Shein, E. V. and L.O. Karpachevskyi (Eds.) Izd. Grif i K, Moscow [in Russian].
6. Tikhonravova, P.I. and N. B. Khitrov (2003). Estimation of Thermal Conductivity in Vertisols of the Central Ciscaucasus Region." Pochvovedenie, 2003. No. 3, 342–351.

“Article is published for the first time”

**Evgeny V. Shein**, professor at Soil Physics and Reclamation Department, Soil Science Faculty  
Moscow State University named M.V.Lomonosov Leninskie Gory 119991 Moscow, Russia.

Professor at Dokuchaev Soil Science Institute, Pyzhevskiy per.,7.

119017, Moscow, Russia.

Tel: +7 903-539-88-19

E-mail: evgeny.shein@gmail.com

**Ahmed Y. Mady**, PhD. Student at Lomonosov Moscow State University, Soil Science Faculty,  
Soil Physics and Reclamation Department, 119991, Moscow Russia.

Assistant Lecture at Ain shams university, Faculty of Agriculture, Soil science Department,  
11241, Cairo, Egypt.

Tel: +7916-990-90-50

E-mail: ahmed\_mady@agr.asu.edu.eg