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UNCERTAINTIES ON FREE VIBRATION OF INHOMOGENEOUS ORTHOTROPIC REINFORCED CONCRETE PLATES

Analyzing the nearly collapsed and broken structures gives a good insight in the possible architectural and engineering design mistakes, faults in the detailing and the mismanagement of the construction by the building contractors. Harmful vibration effects of construction operations occur frequently. The background reviews have demonstrated that the problem of vibration serviceability of long-span concrete floors in buildings is complex and interdisciplinary in nature. In public buildings, floor vibration control is required due to meet Serviceability Limit States that ensure comfort of users of a building. In industrial buildings, machines are often placed on floors. Machines generate vibrations of various frequencies which are transferred to supporting constructions. Precision machines require a stable floor with defined and known dynamic characteristics. In recent years there has been increasing interest in the motion of elastic bodies whose material properties (density, elastic moduli, etc.) are not constant, but vary with position, perhaps in a random manner. Concrete is a non-homogeneous and anisotropic material. Modeling the mechanical behavior of Reinforced Concrete (RC) is still one of the most difficult challenges in the field of structural engineering.

One of the several methods for determining of the dynamic modulus of elasticity of engineering materials is the vibration frequency procedure. In this method, the required variables except modulus of elasticity accurately and certainty determined.

In this research, uncertainly analysis of free vibration of inhomogeneous orthotropic reinforced concrete plates has been investigated. Due to the numerous outputs obtained, software package has been written in Matlab and analysis on the data and drawing related charts have been done.

Key words: Free Vibration, Concrete Plates, Orthotropic and Inhomogeneous Behavior, Uncertainly Analysis.

1. INTRODUCTION

Most structural failures are the result of an error made by one of the people involved in the great number of steps between the original idea and the completion of the final structure. For reinforced concrete construction, mainly inadequate plate designs and over-weight structures are the cause of fatal building and bridge failure and related human victims. Floor vibration affects not only the comfort of the occupants but also sensitive equipment that might be on the floor, especially in industrial and laboratory settings [1]. Excessive floor vibration can even cause some equipment to malfunction. The first known stiffness criterion appeared nearly 180 years ago. Tredgold (1828) wrotethat girders over long spans should be «made deep to avoid the inconvenience of not being able to move on the floor without shaking everything in the room». Traditionally, soldiers «break step» when marching across bridges to avoid large, potentially dangerous, resonant vibration.

Free vibrations of plates attracted investigators nearly two centuries ago (Chladni, 1803; Poisson, 1829). The vibrations of uniform plates have been studied quite extensively already (Gontkevich, 1964; Leissa, 1969). Although the rectangular plates of variable thickness received much less attention than the uniform plates,still, they have been investigated quite intensively.

Leissa et al. 1969 to 1987 compiled some works done in the field of homogeneous and non-homogeneous Plates. An excellent survey of work up to 1987 on vibration of homogeneous isotropic and anisotropic plates of various geometries have been given by Gorman 1982, Timoshenko and Woinowsky 1984, Tomar et al. 1982, 1983, 1984, Shames and Dym 1985. After 1987, studies of homogeneous rectangular orthotropic plates have been carried out by several researchers, to mention a few prominent ones. Gorman 1993, Inman 1994, Chakraverty and Petyt 1999, Fares and Zenkour 1999, Elishakoff 2000, Arenas 2003, Rao 2004, Chen et al. 2004, Yeh et al. 2006, Gupta et al. 2006, Lal and Dhanpati 2007, Kerboua et al. 2007, Reddy 2007, Li et al. 2008 and Civalek 2009, Verhoosel et al. 2011, Hasani Baferani et al. 2011, Hosseini et al. 2011, Shojaee et al. 2012, Thai et al. 2012, Tomasz Wroblewski, et al. 2012 have analyzed the transverse vibrations of non-homogeneous rectangular plates of uniform thickness using boundary characteristic orthogonal polynomials. Poisson's ratio has been assumed to be constant [2], [3].

Natural frequency is the frequency at which a body or structure will vibrate when displaced and then quickly released. This state of vibration is referred to as free vibration. All structures have a large number of natural frequencies; the lowest or «fundamental» natural frequency is of most concern. If a frequency component of an exciting force is equal to a natural frequency of the structure, resonance will occur. At resonance, the amplitude of the motion tends to become large to very large.

A designer must evaluate the critical and the free vibration frequencies of plates. In reality, some of the design parameters in structural analysis may be disregarded which can lead to uncertainties. In this research, uncertainly analysis of free vibration of inhomogeneous orthotropic reinforced concrete plates has been investigated. To do this, sensitivity and uncertainly analysis of free vibration to various parameters such as modulus of elasticity in two direction (orthotropic coefficient), compression/tension forces, length to width ratio, density variation of concrete and inhomogeneous coefficient have been investigated. Studying has been done on a set of concrete plates with and without inhomogeneous properties. The plates sections are generally square or rectangular [4], [5].

2. INHOMOGENEOUS BEHAVIOR OF CONCRETE

Material properties affect the critical values of the free vibration frequency of plates. There are some factors which cause the mechanical factors of concrete in one dimension are not uniform and isotropic in the other one. These factors effect on the concrete elasticity modulus, Poisson coefficient and regular relations of vibration frequency of plates. The material modeling of reinforced concrete consisting generally of three phases: cement mortar, aggregate grains and reinforcing steel bars, is a strong compromise between the structural phenomena and available material parameters. In structural analysis, reinforced concrete materials are modeled as a macroscopically homogeneous material with response influences by each of the phases [6].

2.1 Strength of Concrete

Concrete strength is counted as one of the important parameters for the material properties in reinforced concrete structure design. Using the same mixing, concrete could get different compressive strength results in different situations. Following are some of effective factors on compressive strength of concrete. The compressive strength of concrete depends on some main factors for examples the aggregate grading, aggregate/cement ratio as well as the water/cement ratio. Also depends on some minor factors or site factors for examples grout leakage, poor compaction (the influence of gravity force on concentration of layers and type of concrete compression vibration), segregation, grading limits, poor curing and Chemical attacks such as chlorides, sulfates, carbonation, alkali-silica reaction and acids. Some studies show that humidity and good temperature in concreting after 180 days can increase the concrete strength to 3 times. In those seasons with straight sunshine, the temperature increases and humidity of concrete section decreases 2 or 3 degrees. This factor is important in surface areas of slabs.

2.2 Cracks on Concrete Sections

The most important issues raised regarding serviceability are cracking and deflection of the RC structures. It is obvious that any weakness of the concrete in tension at the time of utilization of a structure will result in cracks in the elements of RC. Although the existence of such cracks does not pose any threat to the structure, if their width is not monitored and controlled, there is a possibility of humidity penetrating the RC together with harmful ions and, consequently, causing the corrosion of the bars. This will cause a sense of insecurity among the users of a structure.

Concrete exhibits a large number of micro-cracks, especially at the interface between coarser aggregates and mortar, even before the application of any external loads. The presence of these micro-cracks has a great effect on the mechanical behavior of concrete, since their propagation (concrete damage) during loading contributes to the nonlinear behavior at low stress levels and causes volume expansion near failure. Many of these micro-cracks are initially caused by segregation, shrinkage or thermal expansion of the mortar. Some micro-cracks may develop during loading because of the difference in stiffness between aggregates and mortar, (Mostofinejad, 2006, Tim Gudmand-Hoyer and Lars Zenke Hansen, 2002).

2.3 Density of Concrete

In practice, with a change in gradation and concrete compaction, the density and the compressive strength of concrete are change (table I). It may happen that with a small change in density and without any external interference, the compressive strength of concrete decrease due to the decrease in density [7].

TABLE I
MATERIAL PROPERTIES AS A FUNCTION OF THE COMPRESSIVE STRENGTH

No	Code	formula	References
1	ACI-2008	$Ec = 4.73\sqrt{f_c'}$.	American Concrete Institute
2	CEB-90	$Ec = 10(f_c' + 8)^{1/3}$.	Euro-International Concrete Committee
3	TS-500	$Ec = 3.25\sqrt{f_c'} + 14$.	Turkish Standard Committee
4	IDC-3274	$Ec = 5.7\sqrt{f_c'}$.	Italian Design Council
5	GBJ-11-89	$Ec = \frac{10^2}{\left[2.2 + \frac{34.7}{f_c'}\right]}$.	Chinese Design Council
6	ABA	$Ec = 5.0\sqrt{f_c'}$.	Iranian Concrete Code
7	Mos-2005	$Ec = 8.3(f_c')^{0.35}$.	Prof. Mostofinejad, Davood [6]

2.4 Static Modulus of Elasticity

A reinforced concrete structure may be subjected to four basic types of actions: bending, axial load, shear and torsion. All of these actions can, for the first time, be analyzed and designed by a single unified theory based on the three fundamental principles of mechanics of materials.

Material properties can be defined through concrete strength and modulus of elasticity as proposed in different national building codes through various formulas for the same values of concrete strength. Modulus of elasticity of concrete is a key factor for estimating the deformation of buildings and members, as well as a fundamental factor for determining modular ratio, *n*, which is used for the design of section of members subjected to flexure. Modulus of elasticity of concrete is frequently expressed in terms of compressive strength. Slope of stress-strain curve is defined as elasticity modulus in concrete.

This modulus relates to the kind of concrete, concrete age and speed in loading, concrete properties and mixing percent and more importantly relates to definition of concrete elasticity modulus. According to table I, the two factors, compressive strength and density, have relations with elasticity modulus. In concreting, by being careful about how to compact the concrete and its completion, the concrete will have much compressive strength. So all the aspects that influence in compressive strength and density have in direct influence in elasticity modulus too.

The compressive strength of steam-cured concrete is not as high as that of similar concrete continuously cured under moist conditions at moderate temperatures. Also, the modulus of elasticity E_c of steam-cured specimens may vary from that of specimens moist-cured at normal temperatures. The modulus of elasticity for concrete is sensitive to the modulus of elasticity of the aggregate and may differ from the specified value. Measured values range typically from 120 to 80 percent of the specified value.

It was concluded that the effect of variation of E_c is twofold: direct effect and indirect effect. The direct effect was discussed earlier as E_c being part of the stiffness matrix of the structure and therefore significantly effecting the natural period and displacement of the structure under dynamic and static loading. The indirect effect shows up in calculating the cracked section moment of inertia of the structure, using higher modulus of elasticity in the elastic analysis will result in a stiffer structure along with a lower displacement demand and therefore less cracking of the concrete. Reduction of cracking of concrete will result in increase in the effective (cracked) moment of inertial [6], [7].

There are various research works available in the literatures for determining sensitivity of modulus of elasticity to vibration of plates. In this study, a procedure was taken into account to investigate the effect of material uncertainties. In the present study, selected seven different design codes were considered in the analyses (table II). Relationships of f'_c and E_c are expressed in MPa and in GPa, respectively. Relationship curves of elasticity Modulus and various concrete strengths for different design codes are shown in Fig. 1.

TABLE II
ELASTIC MODULUS VALUES FOR A GIVEN CONCRETE STRENGTH

REFERENCES	E_c, modulus of elasticity (kg/cm^3) x 10^5			
	f'_c=250 (homogeneous)	f'_c=250 (in-homogeneous) K_c=0.9	f'_c=350 (homogeneous)	f'_c=350 (in-homogeneous) K_c=0.9
ACI-2008	2.34	2.22	2.77	2.63
CEB-90	3.16	3.08	3.45	3.58
TS-500	2.98	2.9	3.28	3.18
IDC-3274	2.82	2.68	3.34	3.17
GBJ-11-89	2.75	2.64	3.09	2.99
ABA	2.48	2.35	2.93	2.78
MOS-2005	2.53	2.44	2.84	2.74

2.5 Dynamic Modulus of Elasticity

Dynamic modulus is consistently higher than static modulus. Static and dynamic moduli follow different mixture behaviors in composite materials such as concrete. It is this difference that may cause dynamic moduli to be higher than static moduli in concrete. Non-destructive dynamic methods can be used to estimate in-place E, but the meaning of the obtained dynamic modulus is uncertain because E_d is known to be different (higher) from that obtained by direct static testing of a cylinder drawn from the structure. Concrete is expected to show a nonlinear dependence between stress and strain, even at low values of deformation caused by quasi-static tests and dynamic tests based on stress-wave propagation [8].

Several attempts have been made to correlate static (E) and dynamic (E_d) moduli for concrete [3], [8]. The simplest of these empirical relations is proposed by Lydon and Balendran [Neville, 1997]:

$$E = 0.83E_d \quad (1)$$

Another empirical relationship for concrete's elastic moduli was proposed by Swamy and Bandyopadhyay and is now accepted as part of British testing standard BS8110 Part2:

$$E = 1.25E_d - 19 \quad (2)$$

where both units of E and E_d are in GPa.

For both lightweight and normal concretes, Popovics suggested a more general relationship between the static and dynamic moduli as a function of the density of the concrete:

$$E_s = kE_d^{1.4}\rho^{-1} \quad (3)$$

where k = 0.23 for units of psi and ρ is density, lbs/ft3 [Popovics, 1975].

2.6 Flexural Stiffness (EI and D)

In defining of the concrete analyzing parameters, the main problem is the choice of stiffness *EI* that reasonably approximates the variations in stiffness due to cracking, creep, and nonlinearity of the concrete stress-strain curve.

It is obvious that due to the non-homogeneous quality of concrete, its tension resistance is not absolutely consistent in any given length. The simple equations for the strength of material can be used to calculate the instantaneous deflection in a steel flexural element. However, calculating the deformation of one beam of RC or a concrete plate encounters the problem of determining stiffness of its section. In fact, in a beam/plate with a steel section, the elasticity module is stable during loading and reaching the yielding limit. Furthermore, under bending, the steel section will present its ultimate strength both in compression and tension, and therefore, its moment of inertia and consequently its bending stiffness will be fixed. However, in a beam/plate with a concrete section

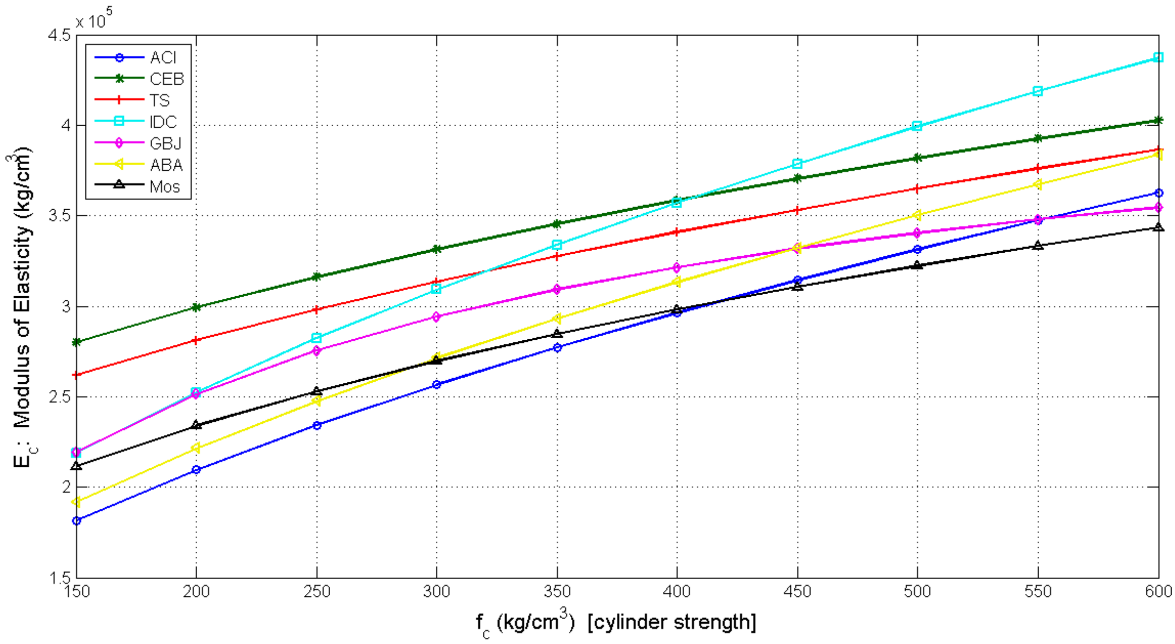

Figure 1. Changes of elasticity Modulus to various concrete strengths for different design codes

both the elasticity module and the moment of inertia will vary during loading due to crackings. In such a case, it is important that the change in the moment of inertia be proportional to the change in the amount of load along the beam/plate.

As previously discussed, calculating the instantaneous deflection faces two major problems: stiffness change in the section of the beam/plate at the time of loading because of tension stiffening, and stiffness change in the length and other various areas of the beam/plate. Not to mention that the elasticity modulus of concrete varies depending on the amount of load. To simplify the calculation of deformations in RC members, the concept of effective flexural stiffness $(EI)_{eff}$ or D is used. If the elasticity modulus of concrete is considered constant-its amount equaling the slope which connects the origin point to the analogous tension point 0.4fc' in the stress-strain-, and if the moment-deformation curve of the member is assumed as a two-line one, the effective moment of inertia can be considered uniform along the member. This effective moment of inertia will be a function of the moment of inertia of the uncracked section (I_{uc}) and the moment of inertia of the uncracked section (I_{cr}), Fig. 2.

According to his broad experimental studies, Branson has proposed the following equation to determine the effective moment of inertia of a beam when calculating deflections.

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^a I_{uc} + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^a\right] I_{cr} \qquad (4)$$

In the above equation, I_{uc} and I_{cr} are the moment of inertia of the uncracked section and the moment of inertia of the uncracked section, respectively; M_{cr} and M_a represent the cracking moment of a section and the maximum flexural moment of the section. Also, a is a constant coefficient which is a=4 for sections under consistent moment and is used to include tension stiffening of concrete in the equation; furthermore, a=3 is proposed by Branson for simply supported beams and is used to include tension stiffening of concrete and also the variations in the stiffness along beams [5], [6].

3. SERVICABILITY LIMIT STATES AND VIBRATIONS OF SLABS

Serviceability limit states in structural engineering occur when the function of the structure is disrupted because of local minor damage, deterioration of structural or nonstructural components, or excessive structural movement. Excessive structural deflections are a main source of unserviceability in buildings. Static deflections may affect the appearance or efficiency of structural and nonstructural elements and mechanical equipment. Dynamic deflections may lead to occupant discomfort and impaired function of sensitive mechanical equipment, especially if resonance occurs [9], [10].

A floor is an integral part of practically every modern industrial, commercial or residential building. As expectations of building users, who are in everyday contact with the floors, rise, so the performance of floor structures in day-to-day service is becoming increasingly important. Most of the knowledge about vibration performance of suspended floors in buildings has been gained during this century. Modern large span floors and light weight floors show a tendency to vibrate under service conditions.

Floor structures are designed for ultimate limit states and serviceability limit state criteria:

- Ultimate limit states are those related to strength and stability;
- Serviceability limit states are mainly related to vibrations and hence are governed by stiffness, masses, damping and the excitation mechanisms.

When considering concrete floors as vibration path caution is required as closely spaced modes of

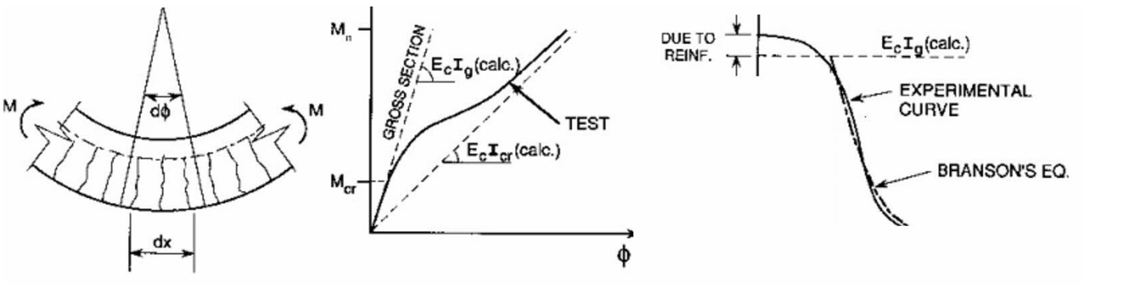

Figure 2. Variation of effective stiffness of the cracked and un-cracked section

vibration tend to occur in orthotropic building floors having repetitive geometry. Typical receivers of floor vibrations are its human occupants or vibration-sensitive equipment. Characterization of humans as floor vibration receivers is probably the most difficult aspect of the floor vibration serviceability problem.

Special attention should be paid in each case to dynamic properties of a designed construction. In public buildings, floor vibration control is required due to meet Serviceability Limit States that ensure comfort of users of a building. The review of the vibration source found that humans are the most relevant excitation of building floors, which is practically impossible to isolate effectively. Moreover, mathematical models simulating human-induced excitation, such as 'near periodic' walking, running or jumping, do exist, but caution is required when using them on lightly damped floor structures. In this case, an assumption of pure resonance may lead to significant overestimation of responses due to inability of humans to generate perfectly periodic forces.

Where occupants can detect vibration in buildings, this may potentially impact on their quality of life or working efficiency. In contrast, people tolerate much higher vibration values in vehicles than in buildings. Individuals can detect building vibration values that are well below those that can cause any risk of damage to the building or its contents. The level of vibration that affects amenity is lower than that associated with building damage.

Vibration serviceability problems were first observed to occur in greater extent in steel joist-concrete composite floors in the 1950s and 1960s. In 1988, when vibration serviceability due to the above mentioned design trends was becoming an issue for a much wider range of floors, including concrete floors, the following three general research tasks requiring further work were identified (Galambos, 1988):

1. To quantify rationally human response to vibrating floors as expressed by the emotions of annoyance and fear,

2. To define rationally the excitation input from human activity on the floor which causes vibration, and

3. To define effectively the structural dynamic model of a real floor in service including determination of more reliable damping.

The American Society of Civil Engineers (ASCE) Standard 7-95 (ASCE, 1995) defines the serviceability limit states (SLS) as: «...conditions in which the functions of a building or other structure are impaired because of local damage, deterioration or deformation of building components or because of occupant discomfort». Occupant discomfort is mainly caused by vibration motion of a building as a whole or of individual building floors. Whereas wind typically excites horizontal vibrations of (tall) buildings, the occupant activities are the main source of floor vertical vibrations (Ellingwood, 1996).

In bridge engineering, dynamic load is common and particular attention should be paid to small and medium span beam bridges that are found along high-speed rail tracks where trains can travel with speeds exceeding 300 km/h.

For the prediction of floor vibration several dynamic floor characteristics need to be determined. Every structure has its specific dynamic behavior with regard to vibration mode shape and duration T[s] of a single oscillation. The natural frequency is the frequency of a free oscillation without continuously being driven by an exciter.

Each structure has as many natural frequencies and associated mode shapes as degrees of freedom. They are commonly sorted by the amount of energy that is activated by the oscillation. Therefore the first natural frequency is that on the lowest energy level and is thus the most likely to be activated [11]–[15].

3.1 Limited Area of Vibration

A floor is a sophisticated, dynamic system. As a continuous structure it has infinite number of modes of vibration (degrees of freedom) but only the first few modes contain almost all the vibration energy of the floor. The first mode (fundamental mode) has the lowest natural frequency, the largest movement, and possesses the lion share (up to 80% in some applications) of the vibration energy. Frequently, quieting this mode alone lowers the overall floor vibration to an acceptable level.

Human activities excite floors at the first few natural frequencies. Such activities usually have forcing frequencies in the range of 1.0 to 3.0 Hz. For instance, walking with a pace of about 2 Hz perturbs a flexible floor at that frequency and its higher order harmonics. When a harmonic of occupants' activities is very close to or matches one of the natural frequencies of the floor, it makes the floor resonate at that frequency causing excessive vibration. As an example, in an office building with reported walking-induced vibration, the average walking pace of the occupants was measured around 2.35 Hz. The first resonant frequency of the floor was measured at about 4.7 Hz. Having the 2nd harmonic of walking exciting the first resonant frequency of the floor caused excessive vibration.

People are known to be very sensitive to floor vibration, e.g. vibration with an amplitude as small as 0.004 inch (0.1 mm) can cause aggravation. Floors that are most disturbing to the occupants often have low resonant frequencies; residential and office building floors having their fundamental frequency usually in the range of 3.5 to 8 Hz, fall in this category. This might be because the natural frequencies of the internal human organs are also in the same frequency range, i.e., 4 to 8 Hz. That is, floor resonance can cause the internal organs of the occupants to resonate resulting in an uneasy and irritating feeling [3], [16]. In table III, some examples of types of vibration have been demonstrated.

Resonance has been ignored in the design of floors and footbridges until recently. Approximately 30 years ago, problems arose with vibrations induced by walking on steel-joist supported floors that satisfied traditional stiffness criteria. Since that time much has been learned about the loading function due to walking and the potential for resonance. More recently, rhythmic activities, such as aerobics and high-impact dancing, have caused serious floor vibration problems due to the resonance [17], [18].

4. UNCERTAINTIES STUDY OF VIBRATION OF CONCRETE PLATES

A rectangular plate of a certain material is assumed, as seen in Figure 3. Forces P_1 and P_2 are applied to it in two main directions and lateral load $P_{(x)}$ can affect it dynamically. The plate is simply supported at its 4 edges, and the general equation of its bending movement is presented in equation, in which L(w) is a function defined in equation, and W is the equation of the plate displacement compared to the X and Y axes. P_1 and P_2 are the compressive and/or tensile forces in directions 1 and 2, respectively, which are applied to the plate at its two ends (Fig. 3). H_0 is the uncracked thickness of the plate, and h is its effective thickness, which is determined through the cracking criteria defined for RC elements. $P_{(x)}$ is the likely transverse force applied to the plate and varies by time, which is shown as t. ω is the angular frequency, which is the desired value to be obtained from equation.

$$L(W) + p_1 \frac{\partial^2 W}{\partial x^2} + p_2 \frac{\partial^2 W}{\partial y^2} + h \rho_0 \varphi(x) \frac{\partial^2 W}{\partial t^2} = p(x) e^{i\omega t} \quad (5)$$

$$L(W) = f(x) \left[D_1^0 \frac{\partial^4 W}{\partial x^4} + \left(D_k^0 + v_2 . D_1^0 + v_1 . D_2^0 \right) \frac{\partial^4 W}{\partial x^2 \partial y^2} + D_2^0 \frac{\partial^4 W}{\partial y^4} \right] + D_1^0 \left[\frac{\partial^2 f}{\partial x^2} \left(\frac{\partial^2 W}{\partial x^2} + v_2 \frac{\partial^2 W}{\partial y^2} \right) + 2 \frac{\partial f}{\partial x} \left(\frac{\partial^3 W}{\partial x^3} + v_2 \frac{\partial^3 W}{\partial x \partial y^2} \right) \right] + D_k^0 \left[\frac{\partial f}{\partial x} . \frac{\partial^3 W}{\partial x^2 \partial y} \right] \quad (6)$$

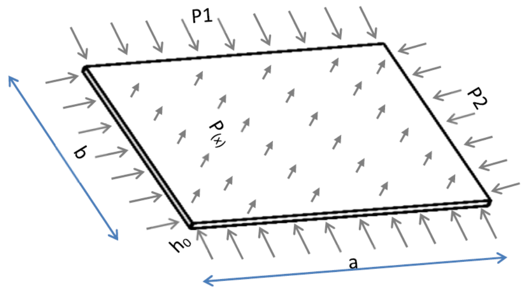

Figure 3. Assumed rectangular plate with applied loads

TABLE III, Examples of types of vibration

Continuous vibration	Impulsive vibration	Intermittent vibration
Machinery, steady road traffic, continuous construction activity (such as tunnel boring machinery).	Infrequent: Activities that create up to 3 distinct vibration events in an assessment period, e.g. occasional dropping of heavy equipment, occasional loading and unloading.	Trains, nearby intermittent construction activity, passing heavy vehicles, forging machines, impact pile driving, jack hammers. Where the number of vibration events in an assessment period is three or fewer this would be assessed against impulsive vibration criteria.

$$E_1 = E_1^0 f(x),\ E_2 = E_2^0 f(x),\ G = G_0 f(x);$$
$$\rho = \rho_0 \psi(x)$$
$$D_1^0 = \frac{E_1^0 h^3}{12(1-\nu_1\nu_2)};\ D_2^0 = \frac{E_2^0 h^3}{12(1-\nu_1\nu_2)};$$
$$D_k^0 = \frac{G_0 h^3}{12} \quad (7)$$

Where D^0_1 and D^0_2 are the flexural stiffness of the plate around direction 1 and 2, respectively, and D^0_k represents its total stiffness. ν_1 and ν_2 are Poisson's coefficient of material in two main directions. In order to solve the vibration equation of the plate, we assume:

$$W(x,y) = V(x,y)e^{i\omega t} \quad (8)$$

Through the placement of equation in equation, equation is obtained:

$$L(V) + p_1 \frac{\partial^2 V}{\partial x^2} + p_2 \frac{\partial^2 V}{\partial y^2} -$$
$$- \omega^2 \rho_0 \psi(x) V - p(x) \quad (9)$$

Where L(v) equals:

$$L(V) = f(x)\left[D_1^0 \frac{\partial^4 V}{\partial x^4} + \right.$$
$$\left. + \left(D_k^0 + \nu_2 . D_1^0 + \nu_1 . D_2^0\right) \frac{\partial^4 V}{\partial x^2 \partial y^2} + D_2^0 \frac{\partial^4 V}{\partial y^4} \right] +$$
$$+ D_1^0 \left[\frac{\partial^2 f}{\partial x^2} \left(\frac{\partial^2 V}{\partial x^2} + \nu_2 \frac{\partial^2 V}{\partial y^2} \right) + \right.$$
$$\left. + 2\frac{\partial f}{\partial x} \left(\frac{\partial^3 V}{\partial x^3} + \nu_2 \frac{\partial^3 V}{\partial x \partial y^2} \right) \right] + D_k^0 \left[\frac{\partial f}{\partial x} . \frac{\partial^3 V}{\partial x^2 \partial y} \right]$$
$$(10)$$

When the plate has simply supported connections at its 4 edges, the boundary conditions for solving the above equation are as the following:

$$V = 0,\quad \frac{\partial^2 V}{\partial x^2} = 0 \quad \{x = 0 \quad x = a\}$$
$$V = 0,\quad \frac{\partial^2 V}{\partial y^2} = 0 \quad \{y = 0 \quad y = b\} \quad (11)$$

As obviously seen, it is not easy to accurately solve equation, which depends on the two variable functions $f_{(x)}$ and $w_{(x)}$. To achieve a solution of the equation to bears less error, approximate solution methods such as the proven and effective Bubnov-Galerkin method will be used. According to the above method:

$$V(x,y) = V_0 \varphi_1(x) \varphi_2(x) \quad (12)$$

In this equation, $\varphi_{1(x)}$ and $\varphi_{2(y)}$ are adapted to the boundary conditions in equation. In the next, for determining of the free frequency, the forced vibration will be ignored, and $P_{(x)}$ will be considered as zero. Therefore, we have equation, and following this, equation:

$$\int_0^a \int_0^b \left[(\varphi_1, \varphi_2) + p_1 \frac{\partial^2 \varphi_1}{\partial x^2} \varphi_2(y) + p_2 \frac{\partial^2 \varphi_2}{\partial y^2} \varphi_1(x) - \right.$$
$$\left. - \omega^2 \rho_0 h \psi(x) \varphi_1(x) \varphi_2(y) \right] \varphi_1(x) \varphi_2(y)\, \mathrm{dxdy} = 0$$
$$(13)$$

$$\omega^2 =$$
$$= \frac{\int_0^a \int_0^b \left[(\varphi_1, \varphi_2) + p_1 \frac{\partial^2 \varphi_1}{\partial x^2} \varphi_2(y) + p_2 \frac{\partial^2 \varphi_2}{\partial y^2} \varphi_1(x) \right] \varphi_1(x) \varphi_2(y)\, \mathrm{dxdy}}{\rho_0 h \int_0^a \int_0^b \psi(x) \varphi_1^2(x) \varphi_2^2(y)\, \mathrm{dxdy}}$$
$$(14)$$

As mentioned before, $f_{(x)}$ and $v_{(x)}$ are variable functions dependent on the surface coordinates of the plate and show the inhomogeneous mechanical characteristics of the plate body. To solve the eigenvalues and determine the angular frequency, we will have to assume an equation for them:

$$f(x) = 1 + \varepsilon \frac{x}{a};\ \psi = 1 + \mu \frac{x}{a};\ \varepsilon \in [0,1];\ \mu \in [0,1]$$
$$\varphi_1(x) = \sin \alpha_m x;$$
$$\varphi_2(y) = \sin \beta_n y \quad \left(\alpha_m = \frac{m\pi}{a};\ \beta_n = \frac{n\pi}{b} \right) \quad (15)$$

Through the placement of equation in equation, we have equation:

$$L(\varphi_1, \varphi_2) = \bar{D}(1 + \varepsilon \bar{x}) \sin \alpha_m x \sin \beta_n y -$$
$$- D_1^0 \varepsilon \left(\alpha_m^3 + \nu_2 \beta_n^2 \alpha_m \right) \alpha^{-1} y . (\cos \alpha_m x . \sin \beta_n y) -$$
$$- 2D_2^0 \varepsilon \left(\beta_m^3 + \nu_1 \alpha_m^2 \beta_m \right) b^{-1} x\, (\sin \alpha_m x . \cos \beta_n y) +$$
$$+ D_k^0 \varepsilon \left(\alpha^{-1} \right) \sin \alpha_n x . \cos \beta_m y\, \alpha_n^2 \beta_m$$
$$(16)$$

$$\bar{D} = D_1^0 \alpha_m^4 + \left(D_k^0 + \nu_2 D_1^0 + \nu_1 D_2^0 \right) \alpha_m^2 \beta_n^2 + D_2^0 \beta_n^4$$
$$(17)$$

Where m=n=1 will activate the main frequency mode and provide the main vibration frequency of the structure. In this paper, frequency sensitivity and frequency change will be investigated in both states where m=n=2 and m=n=3.

$$\omega^2 = \frac{\bar{D}\left(1+0,25\varepsilon\right)-\alpha_m^2 p_1-\beta_n^2 p_2}{\rho_0 h\left(1+0,25\mu\right)} \qquad (18)$$

The above equation shows the angular frequency of the vibration in a rectangular plate which can be inhomogeneous both in terms of density and mechanical characteristics (E and G). At the same time, this plate can also possess orthotropic property $E_1 \neq E_2$. The inhomogeneous property affecting the density of the plate body is defined with the variable μ. If μ =0, it means that the plate body is homogeneous in terms of density variations. Also, the inhomogeneous property affecting the elasticity modulus or shear modulus is defined with the variable ε. If ε=0, the plate body will be considered homogeneous.

In addition to defining the characteristics of inhomogeneous density, mechanical characteristics, and a plate's orthotropic property, equation 12 also provides the angular frequency of a square plate under P_1 and P_2 forces.

To determine the sensitivity of the mentioned variables (inhomogeneity of density, inhomogeneity of mechanical characteristics such as E and G, orthotropic quality, changes in plate dimensions, changes in the ratio of length to width *b/a* and changes in applied forces in two main directions P_2/P_1), an analysis of their sensitivity will be conducted and the results will be presented.

At every stage, the results will be compared to the angular frequency ω_0, which is the angular frequency of the homogeneous isotropic plate without the loads P_1 and P_2 being applied.

Regarding the direct relation between angular frequency and variable *D*,Equation, it is deemed necessary to investigate the uncertainty and sensitivity of other variables to *D*. If the plate is orthotropic, and if the ratio of the elasticity modulus in the main direction E_2/E_1 varies between 1 and 2, the ratio of D_{e2}/D_{e1} will vary between 1 and 1.5. Figure 4 depicts the variation curve of this ratio for the different values of E_2/E_1 and also in various surfaces of the ratio of length to width b/a.

4.1 Investigating the sensitivity of the angular frequency to variations in the ratio of applied loads P_2/P_1

It is assumed that the body of a plate is isotropic and homogeneous, and the angular frequency is indicated with ω_{hi}. The first index (h) and the second index (i) indicate homogeneity and isotropic quality, respectively. In this condition, the ratio of angular frequency to base angular frequency ω_0 equals,

$$\frac{\omega_{hi}^2}{\omega_0^2} = 1 - \frac{\alpha_m^2 P_1 + \beta_n^2 P_2}{D} \qquad (19)$$

In the following, the sensitivity of powered frequency ratio in the applied loads of P_2/P_1 will be analyzed. Equation has a linear relationship with P_2/P_1; therefore, the powered ratio of angular frequency will vary in a linear fashion. Figure 5

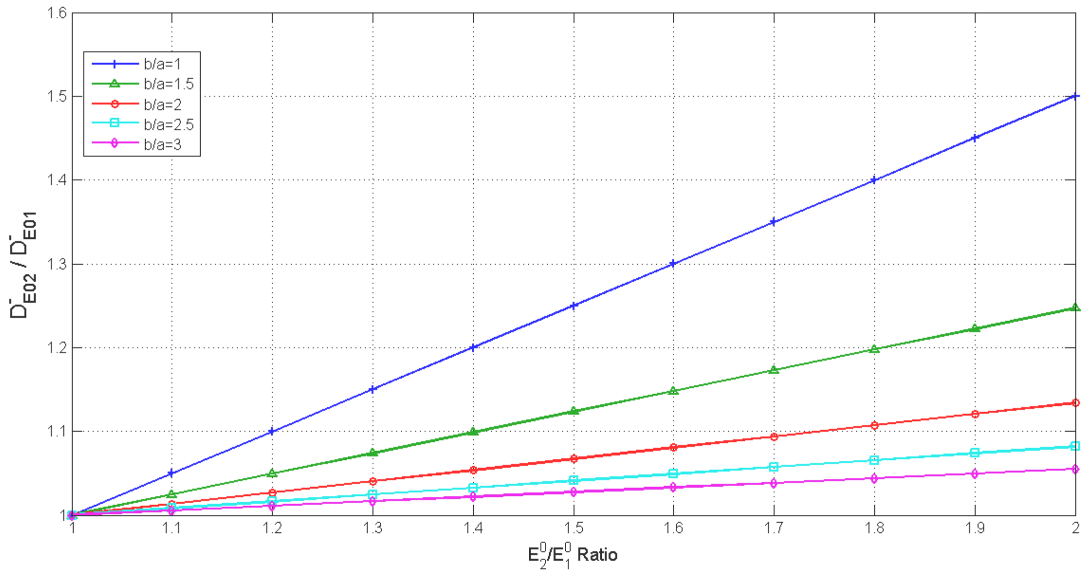

Figure 4. variation of D_{e2}/D_{e1} ratio to E_2/E_1 ratio changes, various b/a

depicts the curve for the variations of the powered angular frequency to different ratios of b/a=1 and b/a=3. As seen in the figure, for the ratios $b/a \geq 2$, the ratio of powered angular frequency by the ratio of load P_2/P_1, equals 0.885 as a constant amount and does not change with the increase in the ratio of length to width, b/a.

4.2 Investigating the sensitivity of the angular frequency to the density inhomogeneity variable

For angular frequency sensitivity to the density inhomogeneity variable, it is assumed that the body of the plate is inhomogeneous in terms of density but homogeneous and isotropic in terms of materials and mechanical characteristic, i.e. the variable ε is assumed 0 and the variable μ is investigated based on this assumption. In this case, angular frequency is represented by ω_{iid}. The first index (i) indicates isotropic quality, and the second one represents inhomogeneity on density.

$$\frac{\omega_{iid}^2}{\omega_0^2} = \frac{D - (\alpha_m^2 P_1 + \beta_n^2 P_2}{D(1+0.25\mu)} \quad (20)$$

The following will analyze the sensitivity of powered angular frequency ratio under the changes in the variable μ (density inhomogeneity). Equation has a linear relationship with the variable μ; therefore, the powered ratio of angular frequency will vary in a linear fashion. Figure 6 depicts the curve for the variations of the powered angular frequency to the density inhomogeneity parameter (μ). The frequency ratio will decrease, at a certain level of load ratio P_2/P_1 and with an increase in μ, for a rectangular plate which is inhomogeneous only in terms of density but is isotropic and homogeneous in terms of mechanical characteristics. This indicates that despite the inhomogeneity of density (with an increasing rate) the plate structure will be flexible and therefore, will have a higher vibration period. Figure 6 depicts this softening in a square structure (b/a=1) for different levels of load ratio (P_2/P_1) and various values of density inhomogeneity factor (μ). The figure indicates that with an increase in either of the variables μ and P_2/P_1, the frequency ratio decreases, and therefore, the structure flexibility increases. Figure 7 depicts the curves for the variations of the powered angular frequency ratio when the plate has a rectangular shape (b/a=2). Comparing the previous two figures, one can observe that if the plate shape changes to a rectangle of b/a=2 from a square, the frequency ratio will decrease as much as 4 percent. Therefore, it can be concluded that changing in other dimensions of the plate, the frequency ratio will not be affected significantly.

4.3 Investigating the sensitivity of the angular frequency to the inhomogeneity variable of mechanical characteristics of materials

A, The plate body is isotropic

It is assumed that the body of the plate is homogeneous in terms of density but inhomogeneous and isotropic in terms of materials and mechanical characteristic, i.e. the variable μ is assumed 0 and the variable ε is investigated based on this assumption. In this case, angular frequency is represented by ω_{ii}. The first index (i) indicates isotropic quality, and the second one represents inhomogeneity on mechanical characteristics.

$$\frac{\omega_{ii}^2}{\omega_0^2} = (1+0.25\varepsilon) - \frac{\alpha_m^2 P_1 + \beta_n^2 P_2}{D} \quad (21)$$

The following will analyze the sensitivity of powered angular frequency ratio under the changes in the variable ε (inhomogeneity of mechanical characteristics of materials). Equation has a nonlinear relationship with the variable ε; therefore, the powered ratio of angular frequency will vary in a nonlinear fashion. Figure 8 depicts the curve for the variations of the powered angular frequency to the inhomogeneity on mechanical characteristics parameter. For a square plate the frequency ratio will increase 25 percent with increasing 100 percent in ε. Figure 8 depicts this softening in a square plate (b/a=1) for values of E_2/E_1 equals to 1.

B, The plate body is orthotropic

It is assumed that the body of the plate is homogeneous in terms of density but inhomogeneous and orthotropic in terms of materials and mechanical characteristic, (E1≠E2) i.e. the variable μ is assumed 0 and the variable ε is investigated based on this assumption. In this case, angular frequency is represented by ω_{io} and equation will use for its variation.

Figure 9 depicts the curve for the variations of the powered angular frequency in value of E_2/E_1 equals to 1.5 (orthotropic quality). Therefore, it can be concluded that changing in values of E_2/E_1 from 1 to 1.5, the frequency ratio not frequency itself, will increase 1.5 percent.

$$\frac{\omega_{io}^2}{\omega_0^2} = 1 - \frac{\alpha_m^2 P_1 + \beta_n^2 P_2}{D\left(1+0.25\mu\right)}, \quad \mu = \varepsilon \quad (22)$$

4.4 Investigating the sensitivity of the angular frequency to the inhomogeneity variable of both mechanical characteristics of materials and density, ε, μ

For angular frequency sensitivity to the inhomogeneity variable of both mechanical characteristics of materials and density, ε, μ, it is assumed that the body of the plate is inhomogeneous in terms of density and in terms of materials and mechanical characteristic, also has orthotropic quality, i.e. the variables μ and ε are investigated based on this assumption. For simplicity, it is assumed that ε=μ. In this case, angular frequency is represented by ω_{io}. The first index (i) indicates inhomogeneity on density and mechanical characteristics, and the second one represents orthotropic quality.

The following will analyze the sensitivity of powered angular frequency ratio under the changes in the variables ε (inhomogeneity of mechanical characteristics of materials) and μ (inhomogeneity of density). Equation has a

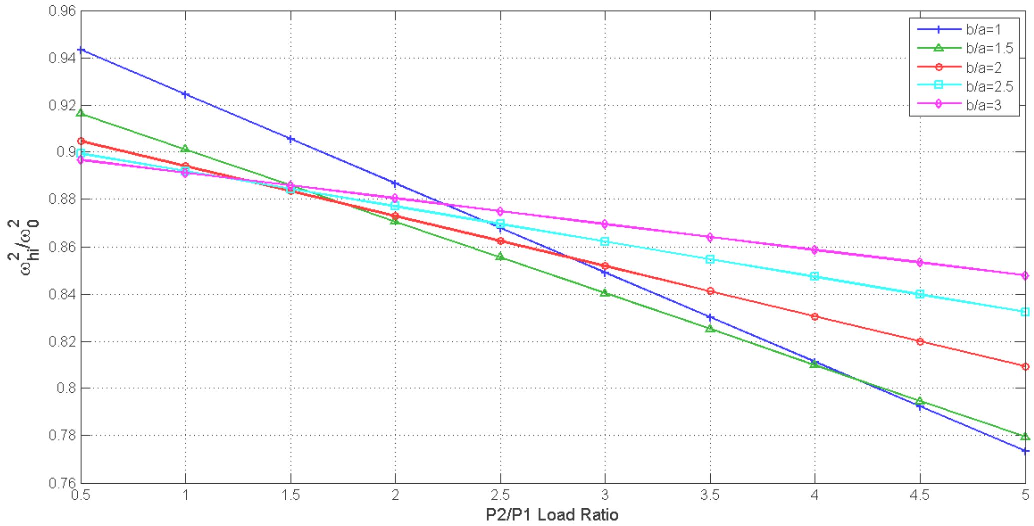

Figure 5. variation of ω_{hi}^2/ω_0^2 ratio to P_2/P_1 ratio changes, various b/a

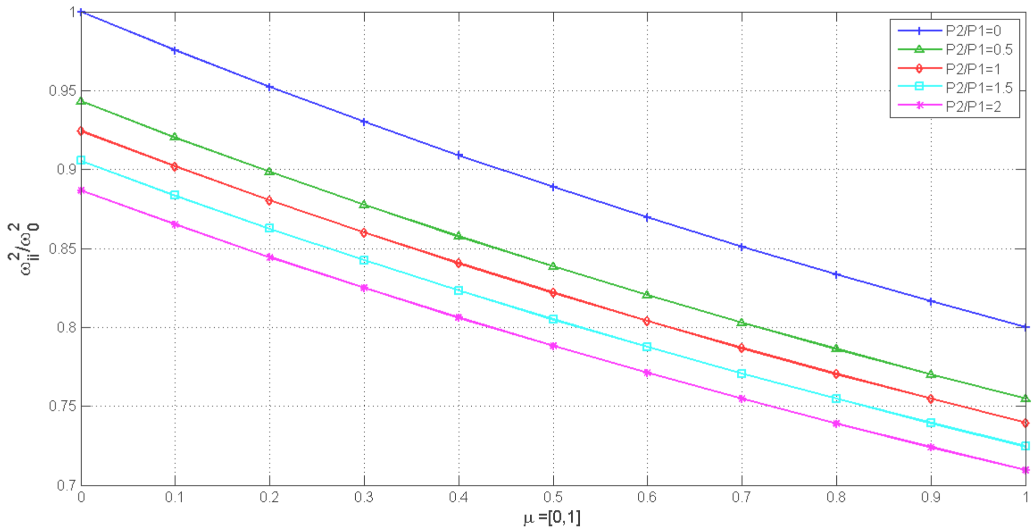

Figure 6. variation of ω_{ii}^2/ω_0^2 ratio to changes of μ=[0,1], various P_2/P_1, b/a=1

nonlinear relationship with the variables μ and ε; therefore, the powered ratio of angular frequency will vary in a nonlinear fashion. Figure 10 depicts the curve for the variations of the powered angular frequency to the density inhomogeneity and inhomogeneity of the mechanical characteristics of the materials parameters, in various ratios of E_2/E_1 and P_2/P_1. For a square plate the frequency ratio will decrease 3% to 4% with increasing in P_2/P_1 for 100%, moduli ratio E_2/E_1 is constant. The figure depicts this softening in a square plate (b/a=1) for various values of P_2/P_1.

4.4 Investigating the sensitivity of the angular frequency to the higher vibration modes, m=n>1

For angular frequency sensitivity in higher vibration modes, (m=n>1) to the variables ε and μ, an analysis of their sensitivity conducted and the results presented in figure 11. The Figure depicts the curve for the variations of the powered angular frequency to the variables ε and μ, in various amount of m=n=1 and m=n=2, witch decreasing in the powered angular frequency with increasing in m=n values. In the other hand, sensitivity of the powered angular frequency on changing the

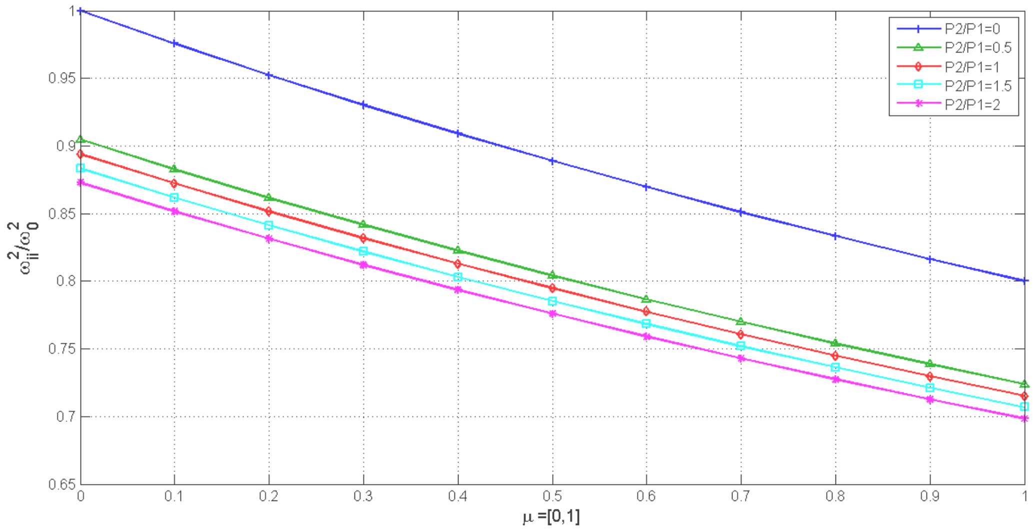

Figure 7. variation of $\omega_{ii}{}^2/\omega_0{}^2$ ratio to changes of μ=[0,1], various P_2/P_1, b/a=2

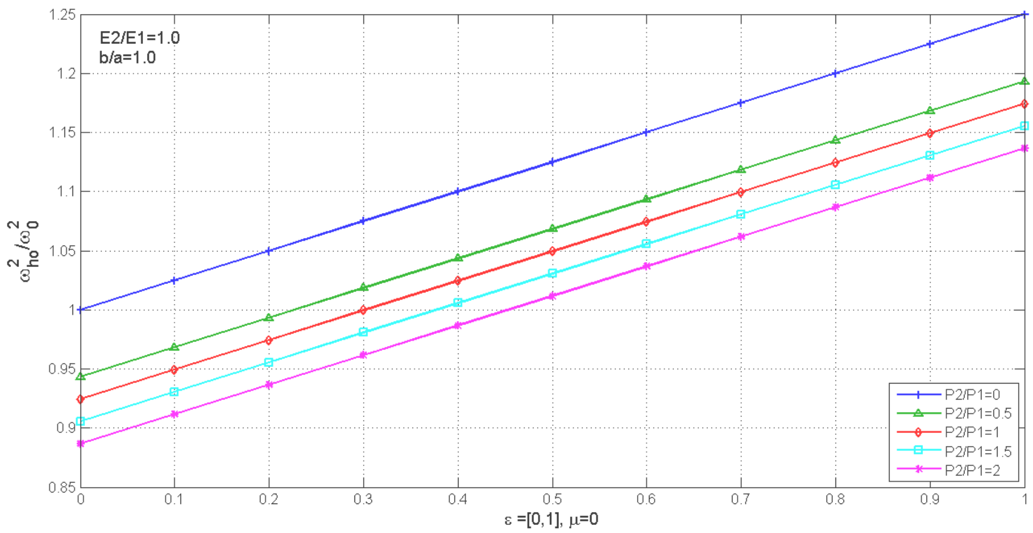

Figure 8. variation of $\omega_{ho}{}^2/\omega_0{}^2$ ratio to changes of μ=[0,1], various P_2/P_1, b/a=1, E_2/E_1=1

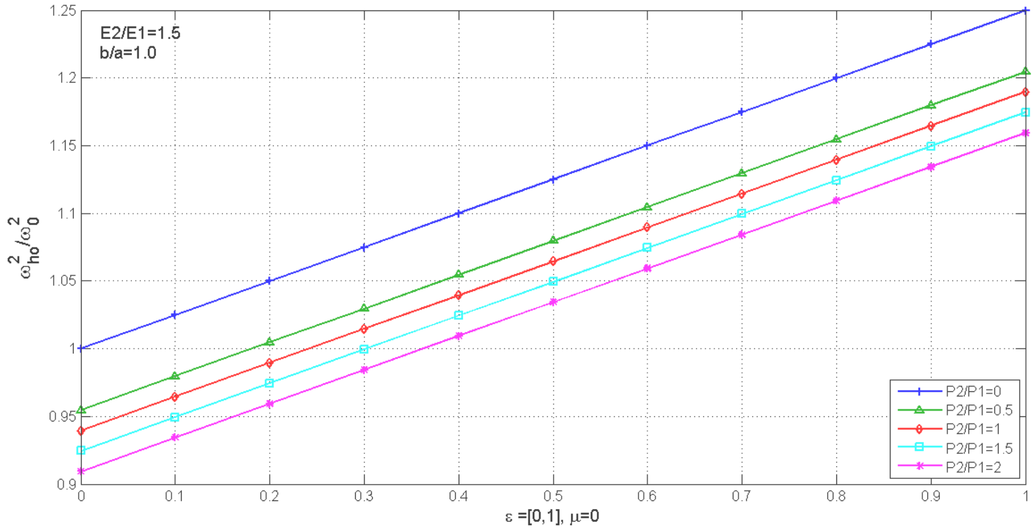

Figure 9. variation of ω_{ho}^2/ω_0^2 ratio to changes of μ=[0,1], various P_2/P_1, b/a=1, E_2/E_1=1.5

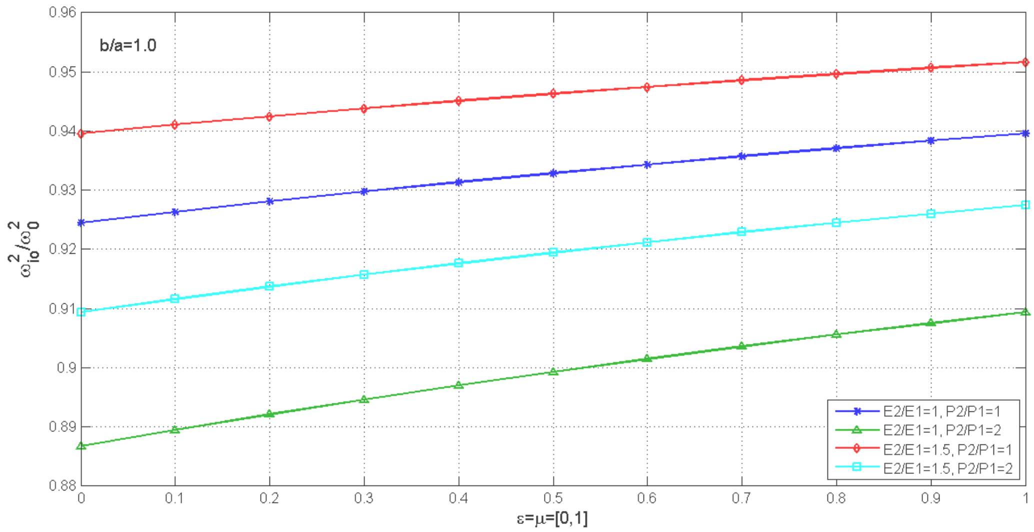

Figure 10. variation of ω_{io}^2/ω_0^2 ratio to changes of ε=μ=[0,1], various P_2/P_1 and E_2/E_1

variables ε and μ , is larger in first vibration mode not in the second and third one.

5. CONCLUSION

One of the several methods for determining of the dynamic modulus of elasticity (DME or E_D) of engineering materials is the vibration frequency procedure. In this method, the required variables except modulus of elasticity accurately and certainty determined. So the dynamic modulus of elasticity is determined with high certainty. Also we know, DME varies with vibration mode and is not constant. Due to the nature of the inhomogeneity, orthotropic quality, cracking and nonlinearity of stress-strain curves of concrete, determining of the dynamic modulus of elasticity of reinforced concrete plates with described method is not easy and has many uncertainties. In uncertainty analysis some variables usually determined as relatively accurate, others are estimated approximately.

Among the variables examined in this study, cross-section geometry, length and width of plate and applied loads to the plate as the first group (with certainty), and variables of mechanical properties including density, elasticity modulus,

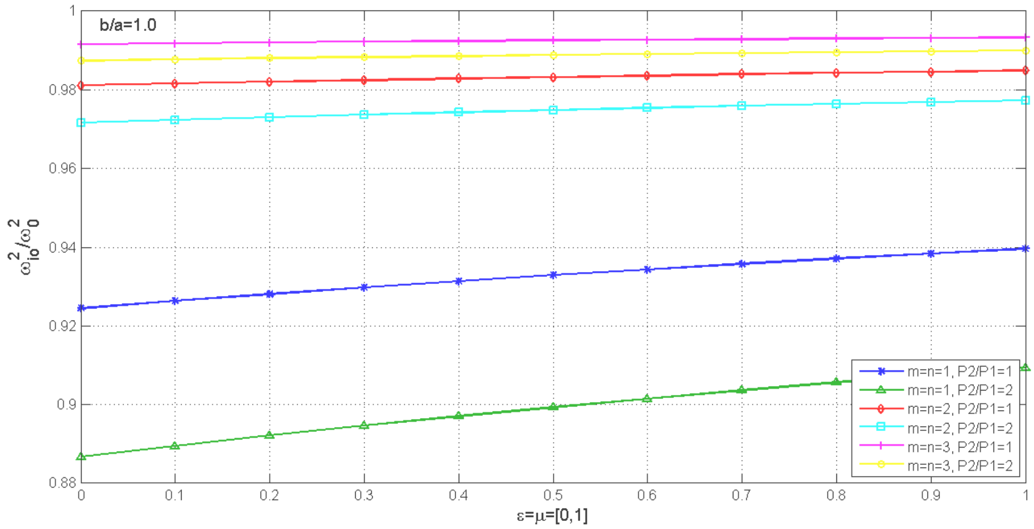

Figure 11. variation of $\omega_{io}{}^2/\omega_0{}^2$ ratio to changes of ε=μ=[0,1], in higher vibration modes

shear modulus, modulus of elasticity of the two main directions, the effective thickness of the plate as the second group (with uncertainties) are located.

Various methods for approximate estimating of the second groups of variables have been presented by various researchers and codes. One of areas of this research is estimating of the frequency errors based on uncertainties of the second group of parameters. For example, in the estimating of the modulus ratio (orthotropic quality) that there is approximately 15% of uncertainty, probability of a few percent of error in determining of the frequency ratio would not be far-fetched. Or if in determining of the inhomogeneity of the mechanical properties to be confronted with a 5% uncertainty, in the desired frequency ratio output, a few percent of error would not be far-fetched.

In this study, the effect of material uncertainties on the free vibration of inhomogeneous orthotropic concrete plates was investigated. Material properties affect the critical values of the free vibration frequency of plates. Material uncertainty was represented by the main important parameters of concrete as concrete strength and elastic modulus. Using the same mixing, concrete could get different Compressive strength results in different situations. In practice, with a change in gradation and concrete compaction, the density and the compressive strength of concrete are change.

To determine the uncertainty and sensitivity of every defined variable (inhomogeneity of density, inhomogeneity of mechanical characteristics of materials such as E and G, orthotropic quality, changes in plate dimensions, changes in the ratio of length to width b/a, and changes in applied forces in two main directions(P_2/P_1)), the sensitivity of the parameters were analyzed and the results presented.

Based on the hyper-geometric solution, numerical values of free vibration frequencies of inhomogeneous orthotropic concrete plates are computed and presented.

18.03.2014

References:
1. Bazant, Zdenelk P., «Structural stability», International Journal of Solids and Structures, no 37, (2000).
2. Krauberger N., et. al., «Inelastic buckling load of a locally weakened reinforced concrete column», Engineering Structures, vol. 34, 278–288, (2012).
3. Department of Environment and Conservation (NSW), «Assessing Vibration: a technical guideline», ISBN 1 74137 812 5, February (2006).
4. Elishakoff, Isaac, «Eigenvalues of Inhomogeneous Structures», CRC Press, (2005).
5. S. BERCZYNSKI, et. al., «Estimation of nonlinear models» parameters of machine tool supporting systems using incomplete vibration test data», Advances in Manufacturing Science and Technology, 35(2011)3, 5-18.
6. Mostofinejad, Davood, «Reinforced Concrete Structures», Arkaneh danesh, Isfahan university, first edition, (2006).
7. L. Shahsavar, Vahid, «Forced vibration of non-uniform orthotropic inhomogeneous rectangular plates clamped in two sides», Theoretical and Applied Mechanics, AZUAC, Azerbaijan University Of Architecture and Construction, Vol. 21, No 1, (2011).

8. ACI-CRC Final Report, «A Study of Static and Dynamic Modulus of Elasticity of Concrete», October (2008).
9. Seema Sharma et. al., «Vibration Analysis of Non-Homogeneous Orthotropic Rectangular Plates of Variable Thickness Resting on Winkler Foundation», Journal of Applied Science and Engineering, Vol. 15, No. 3, pp. 291_300 (2012).
10. Lal, R., Kumar, Y. and Gupta, U. S., «Transverse Vibration of Non-Homogeneous Rectangular Plates of Uniform Thickness Using Boundary Characteristic Orthogonal Polynomials», Int. Jl. of Applied Mathematics and Mechanics, Vol. 6, pp. 93_103 (2010).
11. Sh. Hosseini-Hashemi et. al., «Free vibration of functionally graded rectangular plates using first-order shear deformation plate theory», Applied Mathematical Modelling 34 (2010) 1276–1291
12. Kiran D. Mali, Pravin M. Singru, «Analytical model to determine fundamental frequency of free vibration of perforated plate by using unit step functions to express non-homogeneity», Journal of Vibro Engineering. Sep. (2012). Vol. 14, issue 3. issn 1392-8716.
13. Smith, A.L., et. al. «Design of Floors for Vibrations. A New Approach», SCI Publication P354, Ascot, (2007).
14. Mark R. Svinkin, «Minimizing Construction Vibration Effects, practice periodical on structural design and construction»,©ASCE,MAY (2004)
15. B. Ellingwood and A. Tallin, «Structural Serviceability: Floor Vibrations», journal of Structural Engineering, Vol. 110, No.2, February, (1984), ©ASCE.
16. Pavic and P. Reynolds, «Vibration Serviceability of Long-Span Concrete Building Floors: Part 1 – Review of Background Information», University of Sheffield, Department of Civil and Structural Engineering, (2007).
17. Y. LUO, A. LI, Z. KANG, «Parametric study of bonded steel–concrete composite beams by using finite element analysis», Engineering Structures, 34(2012), 40-51.
18. Shuohui Yin et. al., «Free Vibration Analyses of FGM Thin Plates by Isogeometric Analysis Based on Classical Plate Theory and Physical Neutral Surface», Advances in Mechanical Engineering, hindawi publishing corporation Vol. (2013), http://dx.doi.org/10.1155/2013/634584

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