Kiriakos Amiridis, Basil Psarianos
Lab of Transportation Engineering of National Technical University
of Athens School of Rural and Surveying Engineering
E-mail: kyriakos.amiridis@gmail.com; bpsarian@mail.ntua.gr

APPLICATION OF DIFFERENTIAL GEOMETRY TO SOLVE THE HIGHWAY ALIGNMENT LOCATION PROBLEM

The objective of this paper is to develop a mathematical methodology that allows the design of the 3-d alignment of a road directly in space thus overcoming the conventional approach of studying the three-dimensional road configuration separately and sequentially in its horizontal and vertical alignment.

Initially, the proposed methodology is based on the fact that the road centerline is actually a three-dimensional curve, whereas the road as a whole is in turn a three-dimensional surface. In other words, the geometric road alignment is faced as the mathematical definition of a three-dimensional curve (road centerline) and a three-dimensional surface (road surface).

There are many ways to represent and study a three-dimensional curve. However, in the suggested methodology the road centerline (3-d curve) is selected to be approached as a three-dimensional B-spline curve on the grounds that it has certain significant properties regarding highway engineering which will be described in the discussion that follows. Indeed, the road surface (3-d surface) is selected to be approached as a three-dimensional B-spline surface.

The field of mathematics which is most relevant in studying curves and surfaces is differential geometry. This is why the mathematical background used in the methodology is directly associated with concepts incorporated in differential geometry which are expressed and linked to terms of highway engineering.

The computational part of this approached is attained by expressing the functions in code through the programming language offered by the software Mathematica, whereas the whole process is demonstrated as a case study.

Key words: 3-d highway design, B-Spline curves, B-Spline surfaces, pseudogeodesic curvature, pseudonormal curvature, pseudonormal unit vector, Procrustes transformation.

INTRODUCTION

Conventional road design approach is an a priori approximate design approach on the grounds that it is implemented into two discrete two-dimensional stages, namely: the design of the horizontal alignment and vertical alignment. The resulting 3-d curve is geometrically an arbitrary mathematical curve with unknown design elements in principle.

The conventional highway design process involves separate processing of horizontal and vertical alignments. Due to this fact, some design inconsistencies may result after the construction process. In addition, the design process is accomplished relatively slowly, whereas a trial and error method is also required. Thus, the successful and acceptable design of the alignment of the road depends heavily on the designer's experience and ability.

In this paper a methodology is developed which enables the three-dimensional alignment of the centerline of the road in a fully automatic manner. Specifically, a computational method is developed that uses B-spline curves while fulfilling restrictions with respect to the curve's geometry. These restrictions are in full compliance with the existing highway engineering design policies.

The method proposed here has to be associated with the 2-d geometric features at this stage to provide engineers with the opportunity to become acquainted with the 3-d design parameters. In order to make the application of the proposed methodology feasible, every 3-d element that is used must be linked to its 2-d analogue (for instance, the 2-d horizontal curvature is associated with the 3-d pseudogeodesic curvature). The benefit of the latter is that the accepted values regarding a number of thresholds such as horizontal and vertical curvature, longitudinal slope and superelevation rate, can be directly linked to the three-dimensional design elements.

In the long term it is anticipated that the 3-d vehicle motion will be related to the corresponding 3-d geometric features of a roadway directly, like the Gaussian and mean curvature [1] and 3-d road safety performance functions accordingly defined. This will allow on one side the reduction of the number of performance functions, since for example horizontal and vertical curvature combined with the existing superelevation rate will be addressed in combination in one step and on the other side a correct description of the physical phenomena associated with vehicle motion will be feasible.

LITERATURE REVIEW

Research Methods Of 3-d Alignment In The 20th Century

Recognizing the deficiencies of the conventional two dimensional road design approach a number of researchers tried in the past to address the problem and introduce or propose a new and direct three-dimensional road design process. Brauer [2] was the first researcher who identified the physical properties and meaning of the three-dimensional road curves and referred to their real differential geometric principles and mathematical functions through the necessity of examining the moving Frenet trihedron. Lorenz [3] in another effort suggested a cylindrical barrel approach for obtaining the 3-d road axis within a 3-d route planning process. Many years later and in an effort to introduce a three-dimensional design process Freising [4] suggested a geometric design system using the curve as the 3-d element for the route planning. Scheck's approach [5] involved a gradual dynamic optimization of the route plan in the horizontal and vertical alignment plans. Borgmann [6] examined an interpolation of 3-d fixed points, where the hyperbolic transition curve was used as a flexible 3-d curve resulting from the static properties of continuous stab. Psarianos [7] carried out extensive research into developing a model representation using the 3-d design elements of a straight, a helix and a choroclothoid. In that application the choroclothoid was used as a transition bend between the straight and the helix.

Research Methods Of 3-D Alignment In The 21st Century

Recently Kuhn [8], [9] provided an extensive analytical formulation of the 3-d geometric design methodology of a road based on fixed, coupling, and dialogue elements. Zuo et al. [10] also developed a 3-d road calculation methodology based on computer virtual simulation technology in order to solve the 3-d sight distance problem. Hao et al. [11] integrated visualization in the highway alignment design process in an effort to address efficiently the 3-d road design problem efficiently. Makanae [12] developed a 3-d alignment design system in the virtual space recreated by stereoscopy of aerial photographs. Other 3-d highway design methodologies based on various mathematical functions have been presented by Makanae [13], [14], Karri and Jha [15], Kim and Lovell [16], Jha et al. [17], Kuhn and Jha [18], Karri, et al. [19].

The proposed method in this paper should be seen as a further advancement of the previously mentioned methods. Its main advantage lies in the fact that the roadway is treated as a three-dimensional mathematical surface. This means that all further calculations (e.g. sight distance) and controls (e.g. hydroplaning speed) are much more accurate and realistic. Moreover, due to the 3-d nature of the methodology, any intended road improvement (e.g. to improve sight distance) can be accomplished directly and automatically without taking into account the horizontal and vertical alignment separately.

METHODOLOGY

In this paper a methodology is presented according to which the three-dimensional road design is accomplished in a straightforward manner. Thus the time-consuming trial and error design process is considerably reduced and technological advances are fully utilized. This is accomplished by splitting the curvature vector into its two perpendicular components: the pseudogeodesic and pseudonormal curvature vectors.

The three-dimensional road configuration is studied by applying the theory of differential geometry. According to this theory every three dimensional curve (road centerline) is determined, except from its position in space, by its three-dimensional curvature and torsion functions (fundamental theorem of curves) according to the Frenet-Serret equations [1]. The centerline's absolute position in space is then determined by applying two types of geometric transformations consecutively: the Procrustes transformation which calculates the translation and rotation matrix from one reference system to another and the rigid body transformation, which in turn, calculates the coordinates from one reference system to another. In order to achieve this, the three-dimensional road curvature vector is decomposed into two perpendicular components, namely: the pseudogeodesic and pseudonormal curvature vectors.

As far as the road surface is concerned, it is initially realized as a ruled surface whose directrix [1] is the road centerline (three-dimensional B-spline curve). The rulings of the ruled surface are defined in such a way to be perpendicular to the road centerline. Moreover, the rulings are subjected to a rotation around the road centerline with respect to the horizontal vectors in order to comply with the conventional definition of the superelevation rate of the pavement of the road. Finally, the road surface is approached as a three-dimensional B-spline surface. A detailed description of the process is given in the discussion that follows.

3-d Road Centerline Definition

The methodology suggested, is based on the approach of the road centerline as a three-dimensional B-spline curve. Initially, a temporary road centerline is defined as a three-dimensional B-spline curve acting as a temporary value of an iterative process and afterwards it is modified appropriately. The initial centerline is modified owning to the reason that it must comply with current legislation and certain regulations (i.e. AASHTO, 2011). In general, the final (three-dimensional) road centerline is defined in three steps as follows:

1. STEP: Define Control Points

One very important property regarding B-spline curves is that it can be defined through its control points (de Boor points) [20]. Based on this fact, if the control points of the initial 3-d B-spline curve are defined then the required temporary road centerline is defined as well. The way with which these control are defined depends on whether the road design concerns a new road or the improvement of an existing one.

Specifically, if the design aim is the improvement of an existing road, then the control points correspond to the vertices of the existing horizontal polygonal curve (the orthometric height is also included). However, if the design aims at a new alignment, then the control points correspond to the vertices of any convenient three-dimensional curve like any ground curve or an isocline curve etc.

2. STEP: Calculation of the 3-d Curvatures

The next step relates the 2-d curvature to the pseudogeodesic and pseudonormal curvature vectors. At this point it must be stressed that the pseudogeodesic curvature corresponds to the 2-d curvature of the horizontal plane curves. For example, if the conventional method requires a minimum horizontal radius equal to 200m for a specific road category, then the minimum pseudogeodesic curvature is imposed to be 200m as well. Respectively, the pseudonormal curvature corresponds to the 2-d curvature of the vertical curves (sag or crest vertical curves), meaning that the restrictions concerning the curvature of vertical curves according to the conventional approach are transferred to the pseudonormal curvature directly.

3. STEP: Definition of the Grade Thresholds.

As far as the third and final step is concerned, thresholds (maximum and minimum) are imposed to the longitudinal slope. In terms of differential geometry, the longitudinal slope is expressed as the tangent of the angle which is formed between the unit tangent vector $\bar{t}$ [21] of the road centerline and the horizon. Again as in the second step, the conventional grade value is associated directly to the longitudinal slope of the 3-d curve.

It is worth mentioning that if the road design concerns the improvement of an existing road, then the proposed methodology can be applied directly (without applying the trial and error procedure) as the control points of the B-spline curve correspond to the vertices of the existing horizontal polygonal curve. This means that the number of the control points equals the number of the existing vertices of the horizontal polygonal alignment. In other words, the 3-d coordinates of the control points are known from the beginning meaning that no further digitalization is required.

The three-steps approximation of a road centerline are explicitly discussed in a following paragraph.

Geometric Features of the 3-d Centerline

Fundamental Theorem Of Curves

According to the fundamental theorem of curves [22], the shape of a curve can be directly defined by its curvature and torsion through the Frenet-Serret differential equations (Table 1).

Rigid Body Transformation

The fundamental theorem of curves provides information about the shape of a curve but not about its absolute position in space. In other words, the shape of the centerline of the road may be defined, but this does not mean that it is placed in the correct position as well [23]. To overcome this problem, the shape of the curve is first defined geometrically through the fundamental theorem of curves and then it is placed in the right position in space through the rigid body transformation [24], [25].

Table 1. Fundamental Theorem of Curves Though the Frenet-Serret Equations

Mathematics	Application in the Methodology
definition of the shape of a curve only by its curvature and torsion	definition of the modified shape of the road centerline by imposing thresholds in curvature and maintaining the torsion as it is

Procrustes Rigid Body Transformation

In order to apply the rigid body transformation, the rotation matrix R and the translation vector T must be known [26]. The vector T is actually a 3x1 matrix which shows the distance in which the road centerline must be translated in relation to the axes X, Y and Z. As far as the rotation matrix R is concerned, it shows the way in which the road centerline must be rotated around the axes X, Y and Z. Both matrix R and vector T can be calculated easily through the Procrustes rigid body transformation [27]. This transformation is a straightforward algorithm, which calculates the translation vector and the rotation matrix of a geometric transformation between two reference systems. It is worth mentioning that the Procrustes rigid transformation is a very fast method which needs neither initial starting values nor iteration. The only input required, is the coordinate matrix in the original system xyz and in the target system XYZ (Table 2).

Horizontal Vectors

Horizontal vectors $\vec{h}$ [28] are unit vectors that are perpendicular to the road centerline and at the same moment, perpendicular to the unit vector corresponding to the positive Z axis, $\vec{z} = (0,0,1)$. They are used to analyze the curvature vector into two perpendicular components, namely: the pseudogeodesic and pseudonormal curvature vectors. The analysis of the curvature vector into these two perpendicular components, actually involves the transition from the conventional two-stage design to the 3-d method proposed.

Pseudogeodesic Curvature Vector $\vec{k}_{pg}$

The **pseudogeodesic curvature vector** $\vec{k}_{pg}$ is the vector projection of the curvature vector $\vec{k}$ of a point to its corresponding horizontal vector $\vec{h}$ (Equation 1). Due to the latter, the pseudogeodesic curvature vector has the same direction as the horizontal vector. The prefix «pseudo» is justified by the fact that no reference surface has been taken under consideration, whereas geodesic curvature can be defined only on a surface [29]. It is noted that the pseudogeodesic curvature vector is perpendicular to the unit tangent vector $\vec{t}$.

$$\vec{k}_{pg} = \left(\vec{k} \cdot \vec{h}\right) \cdot \vec{h} \tag{1}$$

Pseudogeodesic Curvature κ_{pg}

The pseudogeodesic curvature vector is a generalization of the curvature vector of a curve in the plane. The pseudogeodesic curvature (Equation 2) is defined as the arithmetic projection of the curvature vector of a point to its corresponding horizontal vector $\vec{h}$ (Table 3).

$$\kappa_{pg} = \vec{k} \cdot \vec{h} \tag{2}$$

Pseudonormal Curvature Vector $\vec{k}_{pn}$

In order to define the **pseudonormal curvature vector** $\vec{k}_{pn}$, the so-called **pseudonormal unit vector** $\vec{N}_p$ must be defined first (Equation 3). $\vec{N}_p$ is the vector, obtained as the vector cross product of the unit tangent vector $\vec{t}$ with the horizontal vector $\vec{h}$ [30]. Eventually, the **pseudonormal curvature vector** $\vec{k}_{pn}$ is defined as the vector projection of $\vec{k}$ on the pseudonormal unit vector $\vec{N}_p$ (Equation 4).

$$\vec{N}_p = \vec{t} \times \vec{h} \tag{3}$$

$$\vec{k}_{pn} = \left(\vec{k} \cdot \vec{N}_p\right) \cdot \vec{N}_p \tag{4}$$

Pseudonormal Curvature κ_{pn}

Concerning the pseudonormal curvature (Equation 5), different limits are imposed to the positive and negative pseudonormal curvature respectively because there are different requirements mainly concerning sight distance and vehicle dynamics.

The pseudonormal curvature is defined as the arithmetic projection of the curvature vector $\vec{k}$ to the pseudonormal unit vector $\vec{N}_p$.

$$\kappa_{pn} = \vec{k} \cdot \vec{N}_p \tag{5}$$

Table 3. Sign of the Pseudogeodesic Curvature κ_{pg}

Mathematics	Application in the Methodology
positive value	right turn of the steering wheel (as the length (stationing) of the road centerline increases)
negative value	left turn of the steering wheel (as the length (stationing) of the road centerline increases)

Table 2. Rigid Body Transformation by Applying the Procrustes Transformation

Mathematics	Application in the Methodology
calculation of the coordinates from one reference system to another	definition of the correct absolute position of the 3-d road centerline by maintaining exactly the same shape of the road centerline

In particular, different limits are given depending on the sign of the pseudonormal curvature (Table 4).

Finally, the link of the main 3-d elements to their 2-d analogue are shown in Table 5 The benefit of the latter is that the accepted values regarding a number of thresholds such as horizontal and vertical curvature, longitudinal slope and superelevation rate, can be directly linked to the three-dimensional design elements.

FIRST APPROXIMATION OF THE ROAD CENTERLINE

The curve, corresponding to the centerline of the road, is approximated as a three-dimensional B-spline curve. The control points of the 3D B-spline curve are exported through the conventional two-stage approach by a specific step length. This 3D curve is temporary and is not imposed to any geometric restrictions (Table 6).

SECOND APPROXIMATION OF THE ROAD CENTERLINE

The road centerline must comply with certain requirements [31]. Thus, the curvature of the centerline is modified accordingly. The shape of the road centerline is re-defined through the fundamental theorem of curves and the Frenet-Serret equations (Table 7). As it has been mentioned before, different limits are given to positive and negative pseudonormal curvature due to several issues such as sight distance. It is noted that the values of pseudogeodesic and pseudonormal curvature correspond to the inverse radius of pseudogeodesic and pseudonormal curvature respectively [30].

THIRD (FINAL) APPROXIMATION OF THE ROAD CENTERLINE

Limits must also be imposed to the longitudinal slope of the second approach of the road leading to the third and final approach of the road centerline (Table 8). Different thresholds are imposed in relation to the sign of the longitudinal slope. Indeed, the longitudinal slope is calculated differently pertaining to its sign (Table 9). If the longitudinal slope is not within the limits, then the unit tangent vector $\vec{t}$ must be rotated by an angle in the hyperplane spanned by the vectors $\vec{t}$ and $\vec{z} = (0,0,1)$. The value of this rotation angle is calculated in such a way so that the longitudinal slope of that specific point is equal to the longitudinal slope limit.

From the above discussion it becomes evident that the values of pseudogeodesic and pseudonormal curvature correspond to the well

Table 4. Sign of the Pseudonormal Curvature κ_{pn}

Mathematics	Application in the Methodology
positive value	3-d curves in sags
negative value	3-d curves over crests

Table 5. Link of the Main 2-d Elements to their 3-d Analogue

2-D Alignment	3-D Alignment
Curvature of the horizontal (plane) curves	Pseudogeodesic curvature
Curvature of the vertical (plane) curves	Pseudonormal curvature
Longitudinal slope	Tangent of the convex angle formed between the unit tangent vector $\vec{t}$ and the vertical of the place.
Superelevation rate	Rotation between the horizontal ruled surface and the road surface (pavement).

Table 6. First Approximation of the Road Centerline

Road centerline	3-d B-spline curve
Control Points (de Boor points)	3-D coordinates corresponding to the horizontal polygonal curve
Spline degree	3 (Cubic Bezier curves)
Absolute position in 3-D space	Correct
Purpose	• Temporary road centerline • Enables calculations of geometrical concepts • Calculation of the coordinate matrix in the target system in the Procrustes transformation

known horizontal and vertical curvatures of the conventional horizontal and vertical alignment accordingly. In this way a road designer can readily associate limiting values of design policies and guides with the proposed 3-d methodology. Consequently the road design outcome, as it will result by implementing the methodology suggested in this paper, will conform totally to the current accepted design policies.

Road Surface

FIRST APPROXIMATION OF THE ROAD SURFACE (Rotated Horizontal Ruled Surface)

The first approximation of the surface of the road is defined as a ruled surface [1] where the directrix corresponds to the third-final approximation of the road centerline. More specifically, the first approximation of the road surface is initially defined by rotating the rulings of a horizontal ruled surface in relation to the superelevation rate. The directrix of the latter horizontal surface is the third (final) approximation of the road centerline, the length its rulings is equal to the semi-length of the road surface. By calculating the superelevation rate, it is possible to define rotation angle required for the transition from the horizontal surface to the first approximation of the road surface. The 3-d coordinates of each point can be calculated through Equation 6. With this approximation it is possible to calculate any geometric element is required on the surface of the

Table 7. Second Approximation of the Road Centerline

Road centerline	3-d B-spline curve
Control Points (de Boor points)	3-d coordinates that occur through the implementation of the fundamental theorem of curves (the curvature limits are taken under consideration).
Spline degree	3 (Cubic Bezier curves)
Absolute position in 3-D space	False. The rigid body transformation must be applied through the Procrustes algorithm where the target system is the matrix containing the 3-d coordinates of the first approximation of the road centerline.
Purpose	Imposition of limits to pseudogeodesic and pseudonormal curvature. These two curvatures are calculated based on the first approximation of the road centerline.
Implementation (shape definition)	Through the fundamental theorem of curves.

Table 8. Third (Final) Approximation of the Road Centerline

Road centerline	3-d B-spline curve
Control Points (de Boor points)	3-d coordinates that occur through the implementation of the fundamental theorem of curves (the longitudinal slope limit is taken under consideration).
Spline degree	3 (Cubic Bezier curves)
Absolute position in 3-D space	False. The rigid body transformation must be applied through the Procrustes algorithm where the target system is the matrix containing the 3-d coordinates of the second approximation of the road centerline.
Purpose	Imposition of the longitudinal slope limits depending on whether the location is an upgrade or downgrade. The longitudinal slope is calculated from the second approximation of the road centerline.
Implementation (shape definition)	Through the fundamental theorem of curves (the change regarding the longitudinal slope leads to a modification of the 3-d curvature).

Table 9. Sign and Calculation of the Longitudinal Slope s

Mathematics		**Application in the Methodology**
positive value	$s = \vec{t} \cdot (0,0,1)$	upgrade
negative value	$s = -\vec{t} \cdot (0,0,1)$	downgrade

road. In addition, the superelevation function is determined by the pseudogeodesic curvature of the directrix, meaning that the superelevation rate is determined straightforward (Table 10).

$$\vec{x}(u,\upsilon)=\vec{\alpha}(u)+\upsilon\vec{\gamma}(u) \quad (6)$$

where $\vec{\alpha}(u)$ and $\vec{\gamma}(u)$ are curves in R^3. Specifically, the curve $\vec{\alpha}(u)$ is the directrix or base curve of the ruled surface and $\vec{\gamma}(u)$ is the director curve [1]. Moreover, the rulings of the ruled surface are straight three-dimensional lines.

It is worth mentioning that all restrictions are taken under consideration concerning the superelevation rate limits, regarding the maximum and minimum permitted limits as well as the upper compound slope limits. The compound slope is the addition of the longitudinal slope s and the superelevation rate q.

SECOND APPROXIMATION OF THE SURFACE OF THE ROAD (3D B-SPLINE SURFACE)

To enable the approach of the surface of the road as an integrated mathematical surface, an interpolation B-spline surface must be applied. The points of the first approximation of the road surface are the ones that operate as the control points of the B-spline surface

Initially, the coordinates corresponding to the left borderline are calculated. Afterwards, the coordinates that are calculated are those that correspond to the u- parametric curve translated to the right by a fixed number in relation to the left border line (as the length of the road centerline increases). This process is repeated until the u-parametric curve corresponds to the right borderline. The step of the discretization of the u-parametric curves is advisable to be applied in such a way in order for the u- parametric curves to pass successively from the left borderline, the road centerline and the right borderline. Through Mathematica, the required coordinates of the surface of the road can be calculated, which will eventually correspond to the control points of the 3D B-spline surface of the road. In this way, the (absolute) three-dimensional coordinates of each point of the surface of the road can be calculated as a function of the curvilinear coordinates u and v.

Finally, on the grounds that the initial points are calculated through a small step discretization, the three-dimensional B-spline surface (second approximation) is defined with a high accuracy in relation to the first approach of the road surface.

IMPLEMENTATION OF THE METHODOLOGY THROUGH THE SOFTWARE MATHEMATICA

In order for the methodology to be applicable and therefore effective it must be implemented in a computational system. In other words, all the commands (e.g. mathematical functions, geometric restrictions, data) are intended to be written in a programming language so that the methodology is applied in a fully automatic manner. All the programs-algorithms required for the automatic computational operation of the methodology are accurately defined by using the software Mathematica, whereas their validity has been checked as well [30]. Eventually, by the achievement of this, the only action required from the highway engineer is to provide Mathematica with the correct input. The numerical example that follows briefly shows the procedure followed by the calculation.

A NUMERICAL EXAMPLE

In the following section the visual results of a case study are presented. In particular, only a section of the road is shown instead of taking into account the whole road so that the validity of the methodology is more easily shown (Figure 1 & Figure 2).

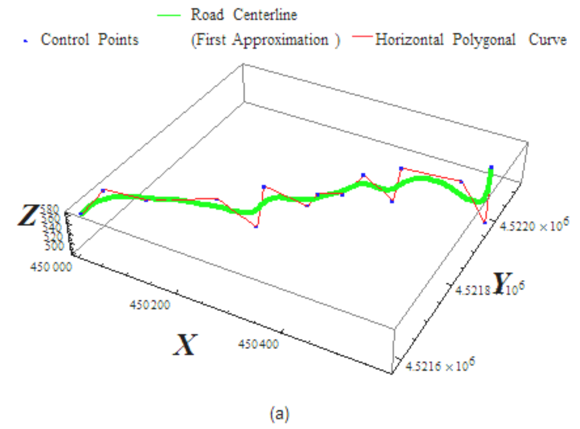
RESULTS AND DISCUSSION

The main advantage of the methodology proposed is that it has the potential of a fully automatic design process including all geometric design controls in a single effort at least for two-lane rural roads. This means that the only action that the user has to consider is to define the valid and applied rules or specifications required for the certain road category design. These specifications are associated with the geometric features of the

Table 10. Connection Between the Pseudogeodesic curvature and the Superelevation Rate q

Pseudogeodesic Curvature	Application in the Methodology	
positive	right turn	Negative q leading to a clockwise rotation of the pavement around the road centerline.
negative	left turn	Positive q leading to a counterclockwise rotation of the pavement around the road centerline.

roadway as these are derived by a design policy (e.g. the upper threshold of curvature or the maximum permitted longitudinal slope). Since the problem is solved in space with all design parameters controlled concurrently the time consuming process



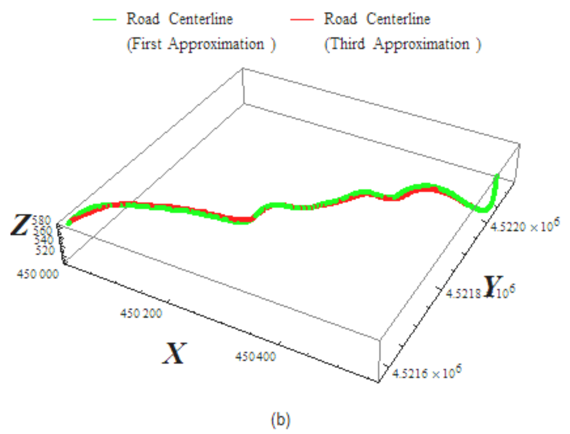


Figure 1. Visual Representation of the Road Centerline: (a) First approximation of the road centerline with its horizontal polygonal curve and vertices (control points) (b) Comparison between the first and third (final) approximation of the road centerline

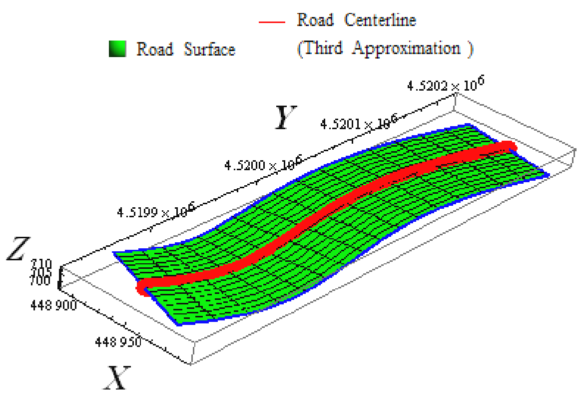


Figure 2. Visual Representation of the Road Surface

of applying a trial and error procedure for satisfying all parameters of the design criteria is no longer required. The fact that the trial and error procedure is no longer required is the significant comparative advantage over all the other methods that have been suggested until now. In other words, the method proposed is fully automated and no user interference is required. Another substantial comparative advantage over the other methods proposed is that it can be applied to the entire road that is under study and not just for a section of it.

CONCLUSIONS AND FUTURE WORKS

This paper describes an application of a new 3-d road alignment methodology that is realistic and takes into account the existing design guides and policies. All the algorithms mentioned in the paper are solved by using the software Mathematica.

Connection of Geometric Elements To Road Safety

In order for the proposed methodology to be more efficient, lower and upper limits must be imposed to geometric aspects that are not yet known. For example, no limits have been imposed to the torsion of the road centerline by the proposed methodology. These limits arise from either natural laws or observation correlations.

From the above discussion it becomes evident that with the proposed methodology a direct 3-d alignment design can be achieved utilizing existing technology and satisfying conventional design criteria as controls as formulated in current design policies. On the grounds of the inclusion of the automatic processing, the proposed methodology can exempt the highway engineer from dealing with tedious tasks such as the trial and error procedure among others. Instead, it gives him the opportunity to concentrate on and address more essential design issues, like design consistency, risk analysis etc.

To recapitulate, as far as the further development of the methodology itself is concerned, more efficient algorithms must be introduced in order for the methodology to be executed faster. In addition, more design concepts and issues of highway engineering should be taken under consideration such as the sight distance and hydroplaning speed for example. The introduction of further concepts will have an impact on the reinforcement of the limitations thresholds that are imposed to the curvature of the road centerline.

Finally a user friendly integrated software must be developed which will carry out the methodology and that will provide the user with all the necessary help required (e.g. function navigator, virtual book) in order to completely embrace its significance and to fully leverage its capabilities.

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Information about authors:

Kiriakos Amiridis, Lab of Transportation Engineering School of Rural and Surveying Engineering National Technical University of Athens graduate student, e-mail: kyriakos.amiridis@gmail.com

Dr. Basil Psarianos, Lab of Transportation Engineering School of Rural and Surveying Engineering National Technical University of Athens professor, e-mail: bpsarian@mail.ntua.gr